Answer on Question #55135 – Math – Statistics and Probability

Consider a random sample (WOR) of two households from a population of households having monthly income (in $) as follows:

Household	1	2	3	4	5
Income	1000	1200	900	1500	1300

Enumerate all possible samples (WOR) of size 2 and show that the sample mean gives an unbiased estimate of population mean.

Solution

Let i be the number of household, X_i is the corresponding income.

The sample mean of pair i, j is given by

$$\frac{X_i+X_j}{2}.$$

All possible samples (WOR) of size 2:

Sample	the sample mean
1 2	1100
1 3	950
1 4	1250
1 5	1150
2 3	1050
2 4	1350
2 5	1250
3 4	1200
3 5	1100
4 5	1400

Population mean is

$$\mu = \frac{1000+1200+900+1500+1300}{5} = 1180. \qquad (1)$$

The sample mean is

$$\frac{\sum \bar{x_i}}{n} = \frac{1100+950+1250+1150+1050+1350+1250+1200+1100+1400}{10} = 1180. \qquad (2)$$

Thus, formulae (1) and (2) show that

$$\frac{\sum \bar{x_i}}{n} = \mu.$$