Answer on Question #55124 – Math – Statistics and Probability
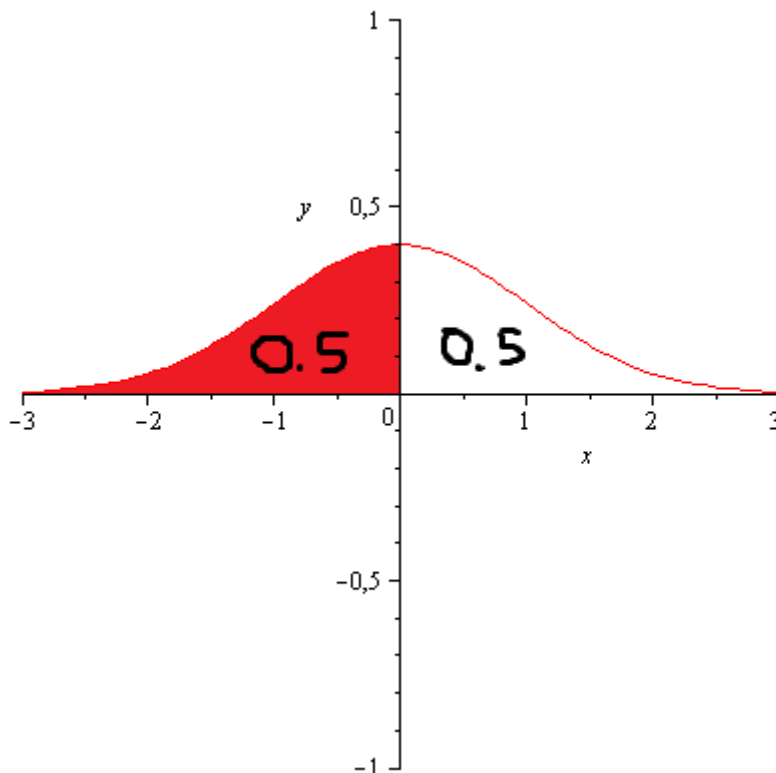
Question

True or False? Justify.

The area under the curve of a standard normal distribution between $-\infty$ and 0 is 0.45.

Solution

The curve of a standard normal distribution has the next form:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, x \in \mathbb{R}.$$

Since $f(-x) = f(x)$, then f is even and its graph is symmetric with respect to y-axis. Then $\int_{-\infty}^{0} f(x)dx = \int_{0}^{\infty} f(x)dx$. Since f is a density of a distribution, we have $\int_{-\infty}^{\infty} f(x)dx = 1$. On the other hand, $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx = 2\int_{-\infty}^{0} f(x)dx$. So we conclude that $\int_{-\infty}^{0} f(x)dx = \frac{1}{2} = 0.5$. The area under the curve of a standard normal distribution between $-\infty$ and 0 is 0.5.

Answer: False.