# Answer on Question #54865 – Math – Algorithms | Quantitative Methods

## Question

If $f(1) = 1, f(3) = 19, f(4) = 49$ and $f(5) = 101$, find the Lagrange's interpolation polynomial of $f(x)$

## Solution

If $f(x_i) = y_i,\ \ i = 0,1,2,\dots,n,$ then Lagrange's interpolation polynomial of $f(x)$ is polynomial of n-th degree in form

$$L(x) = \sum_{j=0}^{n} y_j l_j(x)$$

where

$$l_j(x) = \frac{x - x_0}{x_j - x_0} \cdot \frac{x - x_1}{x_j - x_1} \dots \frac{x - x_{j-1}}{x_j - x_{j-1}} \cdot \frac{x - x_{j+1}}{x_j - x_{j+1}} \dots \frac{x - x_n}{x_j - x_n}$$

We have $x_0 = 1, x_1 = 3, x_2 = 4, x_3 = 5$ and $y_0 = 1, y_1 = 19, y_2 = 49, y_3 = 101$

$$l_0(x) = \frac{x-3}{1-3} \cdot \frac{x-4}{1-4} \cdot \frac{x-5}{1-5} = -\frac{1}{24}(x-3)(x-4)(x-5) = -\frac{1}{24}(x^3 - 12x^2 + 47x - 60)$$

$$l_1(x) = \frac{x-1}{3-1} \cdot \frac{x-4}{3-4} \cdot \frac{x-5}{3-5} = \frac{1}{4}(x-1)(x-4)(x-5) = \frac{1}{4}(x^3 - 10x^2 + 29x - 20)$$

$$l_2(x) = \frac{x-1}{4-1} \cdot \frac{x-3}{4-3} \cdot \frac{x-5}{4-5} = -\frac{1}{3}(x-1)(x-3)(x-5) = -\frac{1}{3}(x^3 - 9x^2 + 23x - 15)$$

$$l_3(x) = \frac{x-1}{5-1} \cdot \frac{x-3}{5-3} \cdot \frac{x-4}{5-4} = \frac{1}{8}(x-1)(x-3)(x-4) = \frac{1}{8}(x^3 - 8x^2 + 19x - 12)$$

Hence $L(X) = l_0(x) + 19l_1(x) + 49l_2(x) + 101l_3(x) =$

$$= -\frac{1}{24}(x^3 - 12x^2 + 47x - 60) + \frac{19}{4}(x^3 - 10x^2 + 29x - 20) - \frac{49}{3}(x^3 - 9x^2 + 23x - 15) + \frac{101}{8}(x^3 - 8x^2 + 19x - 12) = x^3 - x^2 + 1$$

**Answer:** $\boldsymbol{L(x)} = x^3 - x^2 + 1$ .