Answer on Question #54400– Math – Algebra

Question:

The mass of a particular substance is known to grow exponentially at a rate of 12.5% per week. Its initial mass was 34 grams and, after t weeks, it weighed 143 grams. The equation modeling this growth is 34×1.125 t = 143. Use the method of taking logs to solve this equation for t, giving your answer correct to the nearest week. (Your answer should be a number, without units)

Solution:

Definition of the general exponential function:

We say P is an exponential function of t with base a if

$$P = P_0 a^t, \qquad (1)$$

where P_0 is the initial quantity (when t = 0) and a is the factor by which P changes when t increases by 1. If a > 1, we have exponential growth; if 0 < a < 1, we have exponential decay.

According to the statement of the problem we have

$$143 = 34 \cdot (1.125)^t. \qquad (2)$$

Dividing both sides of the equation (2) by "34" we obtain

$$\frac{143}{34} = (1.125)^t. \qquad (3)$$

Now, we apply the natural logarithm "*ln*" to both sides of the equation (3) and use the formula

$$ln A^B = B \cdot ln A. \qquad (4)$$

Namely,

$$ln\left(\frac{143}{34}\right) = ln((1.125)^t) = t\,ln(1.125).$$

Hence, we find *t:*

$$t = \frac{ln\left(\frac{143}{34}\right)}{ln(1.125)} = \frac{1.437}{0.118} = 12.178 \approx 12. \qquad (5)$$

Answer: 12 weeks.