Answer on Question #53984 – Math – Trigonometry

Question

If $t = \tan\left(\frac{x}{2}\right)$

then

$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+t^2}}, \qquad \sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{1+t^2}}, \qquad 0 \le x \le \pi.$$

Solution

If $0 \le x \le \pi$, then $0 \le \frac{x}{2} \le \frac{\pi}{2}$ and

$$\tan\left(\frac{x}{2}\right) \ge 0, \qquad \cos\left(\frac{x}{2}\right) \ge 0, \quad \sin\left(\frac{x}{2}\right) \ge 0.$$

Rewrite

$$\frac{1}{\sqrt{1+t^2}} = \frac{1}{\sqrt{1+\left(\tan\left(\frac{x}{2}\right)\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)^2}} = \frac{1}{\frac{\sqrt{\left(\cos\left(\frac{x}{2}\right)\right)^2+\left(\sin\left(\frac{x}{2}\right)\right)^2}}{\sqrt{\left(\cos\left(\frac{x}{2}\right)\right)^2}}} =$$

$$= \frac{1}{\frac{\sqrt{1}}{\cos\left(\frac{x}{2}\right)}} = \cos\left(\frac{x}{2}\right).$$

Thus, we have $\frac{1}{\sqrt{1+t^2}} = \cos\left(\frac{x}{2}\right)$.

Next,

$$\frac{t}{\sqrt{1+t^2}} = t \cdot \frac{1}{\sqrt{1+t^2}} = \tan\left(\frac{x}{2}\right) \cdot \cos\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} \cdot \cos\left(\frac{x}{2}\right) = \sin\left(\frac{x}{2}\right).$$

So

$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+t^2}}, \qquad \sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{1+t^2}}, \qquad 0 \le x \le \pi$$

where

$$t = \tan\left(\frac{x}{2}\right).$$