Answer on Question #53934 – Math – Analytic Geometry

Question

Find the center, vertices, and foci of the ellipse with equation $2x^2 + 7y^2 = 14$.

Solution

We transform the original equation in the next way: $\frac{x^2}{7} + \frac{y^2}{2} = 1 \Leftrightarrow \frac{x^2}{(\sqrt{7})^2} + \frac{y^2}{(\sqrt{2})^2} = 1$. So we have: $a = \sqrt{7},\ b = \sqrt{2}$. Using the fact that canonical equation of ellipse has the form $\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$ where $(x_0;\ y_0)$ is the center, we conclude that the ellipse $\frac{x^2}{(\sqrt{7})^2} + \frac{y^2}{(\sqrt{2})^2} = 1$ has the center at the point $(0;0)$. The vertices are $A(-\sqrt{7};0), B(0;\ \sqrt{2}), C(\sqrt{7};0), D(0;\ -\sqrt{2})$. The focal length is equal to $c = \sqrt{a^2 - b^2} = \sqrt{7-2} = \sqrt{5}$ so the foci are $F_2(-\sqrt{5};0)$ and $F_1(\sqrt{5};0)$.

Answer: Center: $(0;0)$;

vertices: $(-\sqrt{7};0), (0;\ \sqrt{2}), (\sqrt{7};0), (0;\ -\sqrt{2})$;

foci: $(-\sqrt{5};0)$ and $(\sqrt{5};0)$.