Answer on Question #53781 – Math – Algebra

Let $a > b > 0$. Find the least value of $a + \frac{1}{b}(b - a)$.

Solution

$a + \frac{1}{b}(b - a) = a + 1 - \frac{a}{b}$ does not have the least value, because $a > b$, a is not bounded from above.

From inequalities

$$b(a-b) < \frac{a^2}{4}$$

$$\frac{1}{b(a-b)} > \frac{4}{a^2}$$

$$-\frac{1}{b(a-b)} < -\frac{4}{a^2}$$

it follows that

$$\frac{a+1}{b(b-a)} = -\frac{a+1}{b(a-b)} < -\frac{4}{a^2} \cdot (a+1)$$

Function $\frac{4(a+1)}{a^2} = \frac{4}{a} + \frac{4}{a^2}$ does not attain global extremums, therefore $\frac{a+1}{b(b-a)}$ does not have the least value either.