Answer on Question #53694 – Math – Analytic Geometry

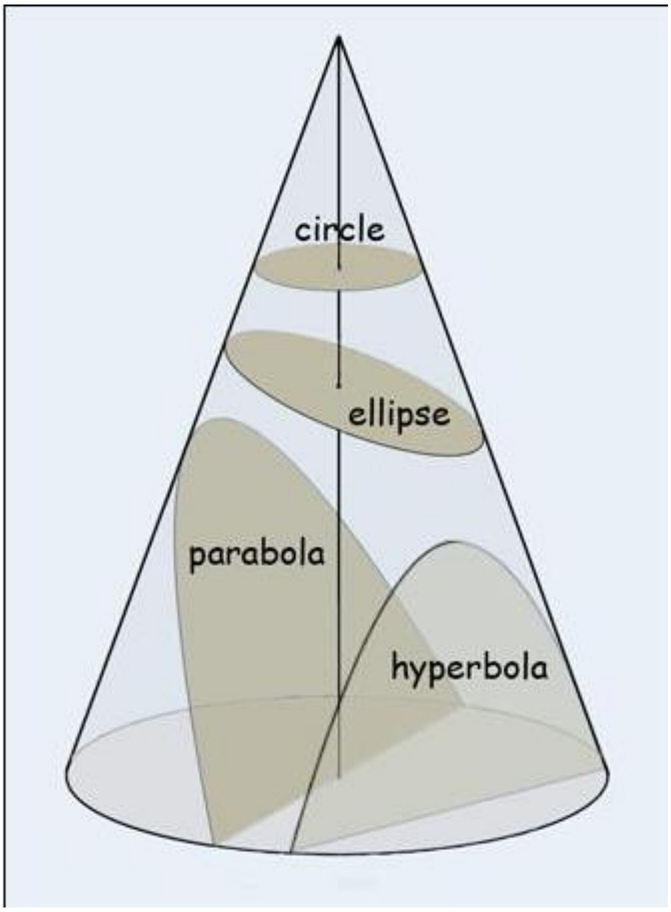
Question

How conic sections are generated. From your surrounding give four examples of each of circle, parabola, ellipse and hyperbola. Prepare a list of their different informations.

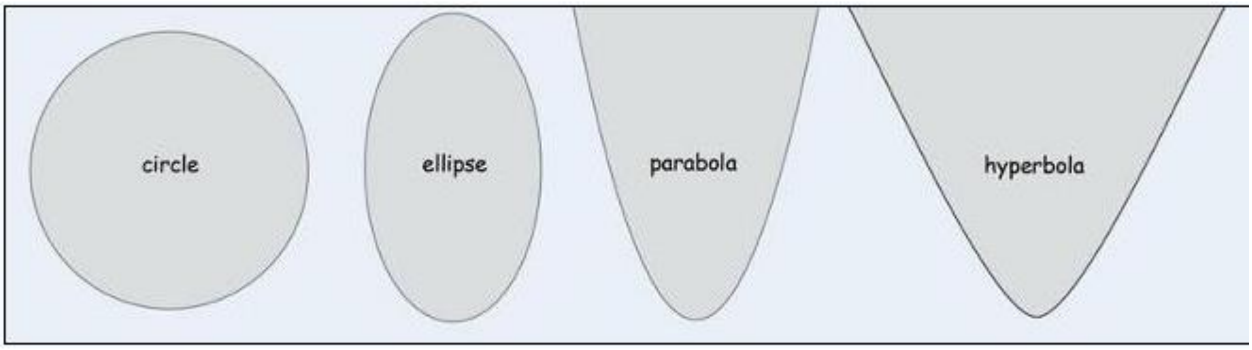
Solution

Conic Sections

A *section* is the surface or outline of that surface formed by cutting a solid figure with a plane. If the solid figure is a right circular cone, the resulting curve is called a *conic section*. The diagram below shows such a cone, formed by rotating a diagonal line around a vertical axis so that the axis, the diagonal and a horizontal line, connecting the two, form a right triangle. Four planes are shown, cutting through the cone at various angles, producing the curves shown in the following diagram. The intersection of each plane with the cone forms a conic section. The kind and shape of the conic section is determined by the angle of intersection of the plane with the axis and surface of the cone.



Angled view of a cone, with conic sections produced by cutting the cone at different angles. Cutting at right angles to the axis produces a circle. Cutting at less than a right angle to the axis but more than the angle made by the side of the cone produces an ellipse. Cutting parallel to a side of the cone produces a parabola. Cutting more nearly parallel to the axis than to the side produces a hyperbola (the hyperbola in the diagram represents a cut parallel to the axis of the cone).



View from above of, from left to right, a circle, an ellipse, a parabola and a hyperbola. A circle is a smooth, uniform curve, while an ellipse is "stretched" out along one axis, and "compressed" along the perpendicular axis. Circles and ellipses are closed curves, while parabolas and hyperbolas are open curves. For parabolas the two arms are parallel to each other at infinity, but for hyperbolas the two arms make an angle with each other even at infinity.

The general equation that covers all conic sections is

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

A conic section may more formally be defined as the locus of a point *P* that moves in the plane of a fixed point *F* called the focus and a fixed line $d$called the conic section directrix (with *F* not on *d*) such that the ratio of the distance of *P* from *F* to its distance from *d* is a constant *e* called the eccentricity. If *e*=0 $e=0$, the conic is a circle, if 0<*e*<1, the conic is an ellipse, if *e*=1, the conic is a parabola, and if *e*>1, it is a hyperbola.

A conic section with conic section directrix at *x*=0, focus at (*p*,0), and eccentricity *e*>0 has Cartesian equation

$$y^2 + (1 - e^2)x^2 - 2\,p\,x + p^2 = 0 \qquad (1)$$

(Yates 1952, p. 36), where *p* is called the focal parameter. Plugging in *p* gives

$$y^2 + (1 - e^2)x^2 - \frac{2\,a\,(1 - e^2)}{e}\,x + \frac{a^2\,(1 - e^2)^2}{e^2} = 0, \qquad (2)$$

for an ellipse,

$$y^2 = 4\,a\,(x - a), \qquad (3)$$

for a parabola, and

$$y^2 + (1 - e^2)x^2 - \frac{2\,a\,(e^2 - 1)}{e}\,x + \frac{a^2\,(e^2 - 1)^2}{e^2} = 0 \qquad (4)$$

for a hyperbola.

The polar equation of a conic section with focal parameter p is given by

$$r = \frac{p\,e}{1 + e\cos\theta}.$$