Answer on Question #53692 – Math – Analytic Geometry

Question

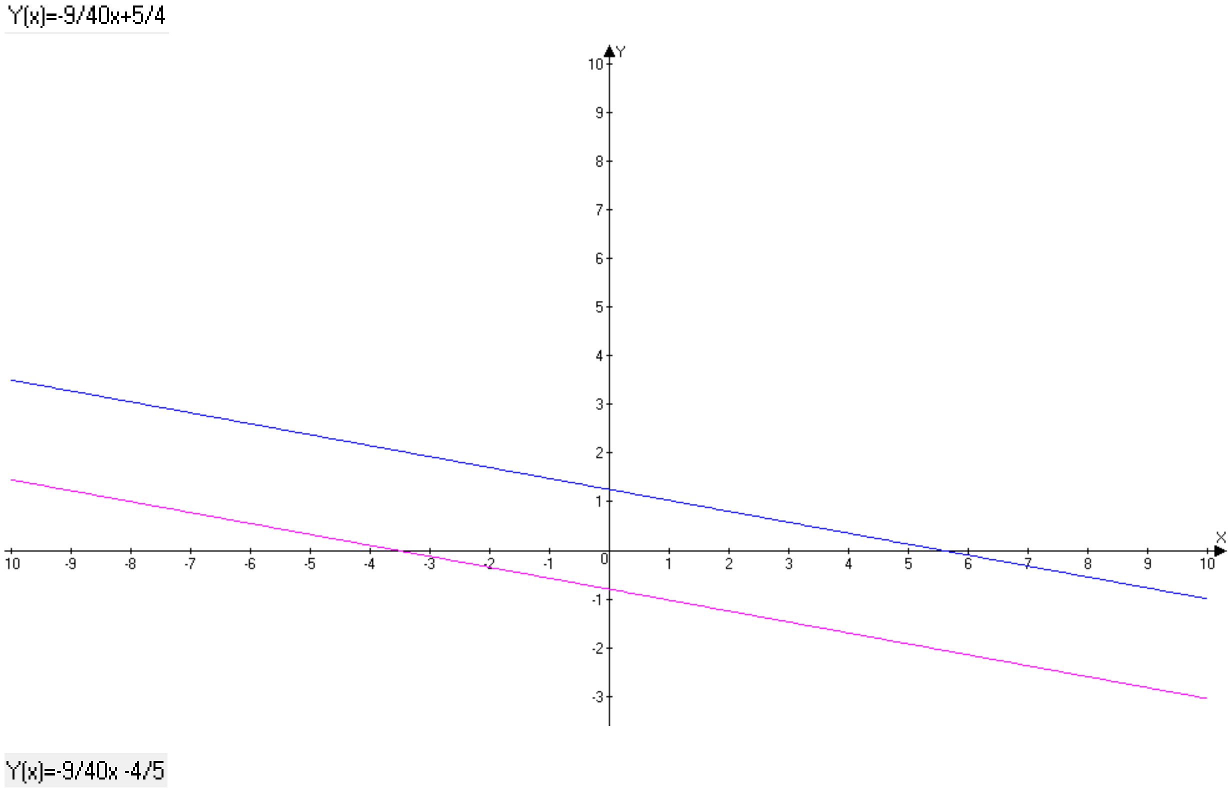
Base of an equilateral triangle lies along the line 9x+40y-50=0 and its vertex opposite to the base lies on the line 9x+40y+32=0. Find the length of the side of the triangle and also find its area.

Solution

$y = -\frac{9}{40}x + \frac{5}{4}$ is equation of the first line.

$y = -\frac{9}{40}x - \frac{4}{5}$ is equation of the second line.

We have the slope of the first line = the slope of the second line = $-\frac{9}{40}$. This means that we have two parallel lines, because lines with the same slope are parallel.So position of an equilateral triangle is not fixed, but the length of the side and its area do not change.



$Ax + By + C_1 = 0$ is equation of the first line

$Ax + By + C_2 = 0$ is equation of the second line

Distance between two parallel lines is $h = \frac{|C_2 - C_1|}{\sqrt{A^2+B^2}}$, hence $h = \frac{32+50}{\sqrt{81+1600}} = 2$;

Length of the side of the equilateral triangle is $a = \frac{2\sqrt{3}}{3}h = \frac{2\sqrt{3}}{3} \cdot 2 = \frac{4\sqrt{3}}{3}$ (units) ;

Area of the equilateral triangle is $S = \frac{ah}{2} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\left(\frac{4\sqrt{3}}{3}\right)^2 = \frac{\sqrt{3}}{4}\,\frac{16\cdot 3}{9} = \frac{4\sqrt{3}}{3}$ (square units).