Answer on Question #53688 – Math – Algebra

What happens to the graph of $f(x) = ax^2+bx+c$ as

(a) a changes while b and c remains fixed?

(b) b changes while a and c remains fixed, $a\neq 0$

(c) c changes while a and b remains fixed, $a\neq 0$

Solution

(a) **a** changes while **b** and **c** remain fixed.

For positive values of 'a' the parabola opens upward.

For negative values of 'a' the parabola opens downward.

If 'a' gets larger, the parabola becomes thinner (narrower), closer to its line of symmetry.

If 'a' gets smaller, the parabola becomes thicker (wider), further from its line of symmetry.

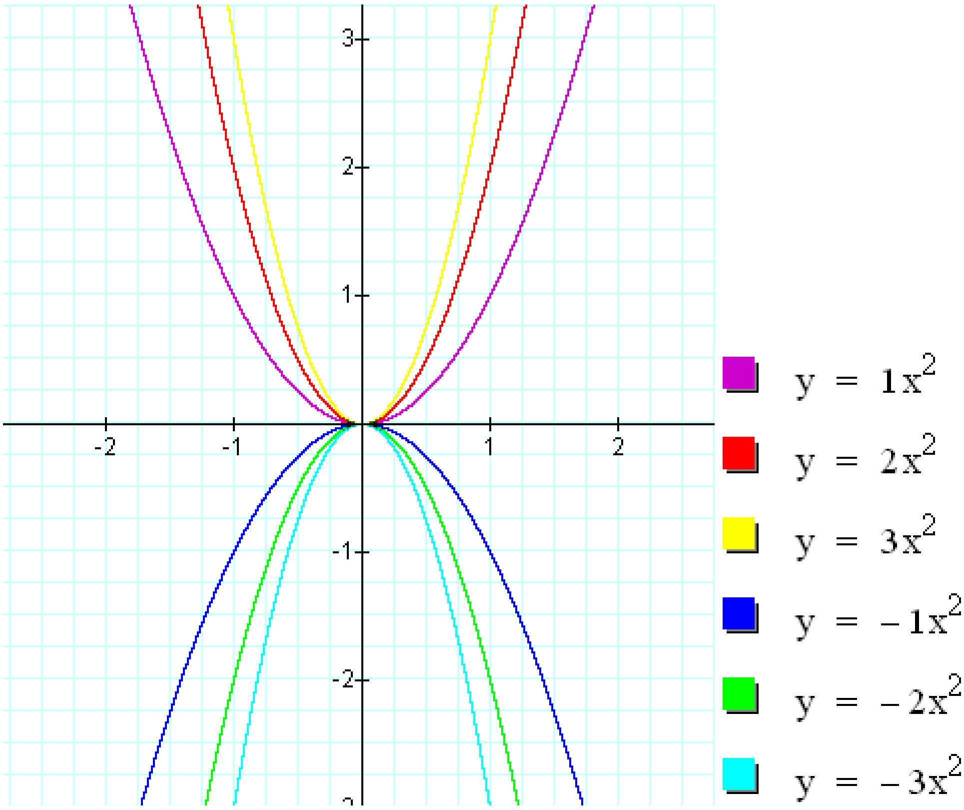

Fig.1

The change of the *a* has the impact on the shape of the parabola. As *a* varies and approaches to 0, the parabola turns to be the straight line. This suggests that the straight line is the limit as *a* approaches 0 from either side.

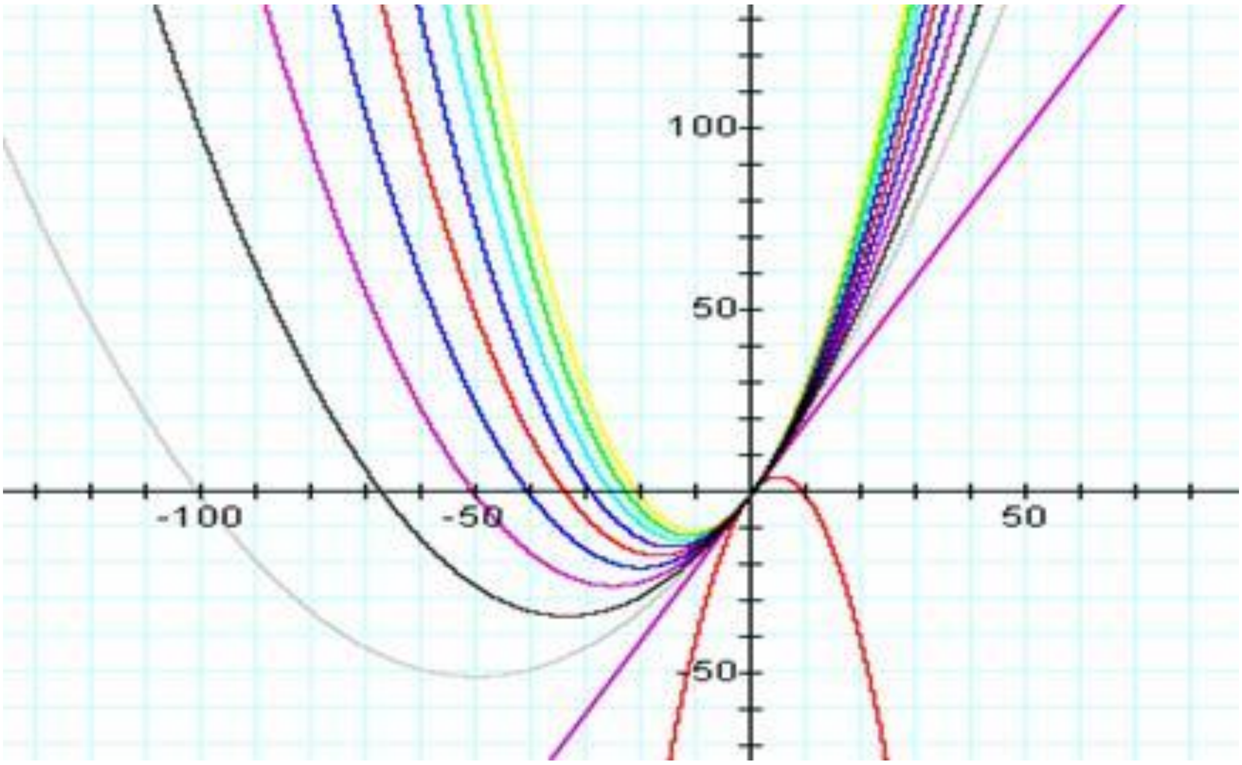

Fig.2

(b) b changes while **a** and **c** remains fixed**, a≠ 0**

If the coefficient b changes while a and c remain fixed, then the x-coordinate of the vertex $x=-b/2a$ changes, and the y-coordinate

$$y = ax^2 + bx + c = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c = \frac{b^2}{4a} - \frac{b^2}{2a} + c =$$

$$= -\frac{b^2}{4a} + c = -\frac{b^2-4ac}{4a} = -\frac{D}{4a}$$

of the vertex changes.

While a and c remain fixed, parabola goes through the point $(x, y) = (0, c)$ at different values of b.

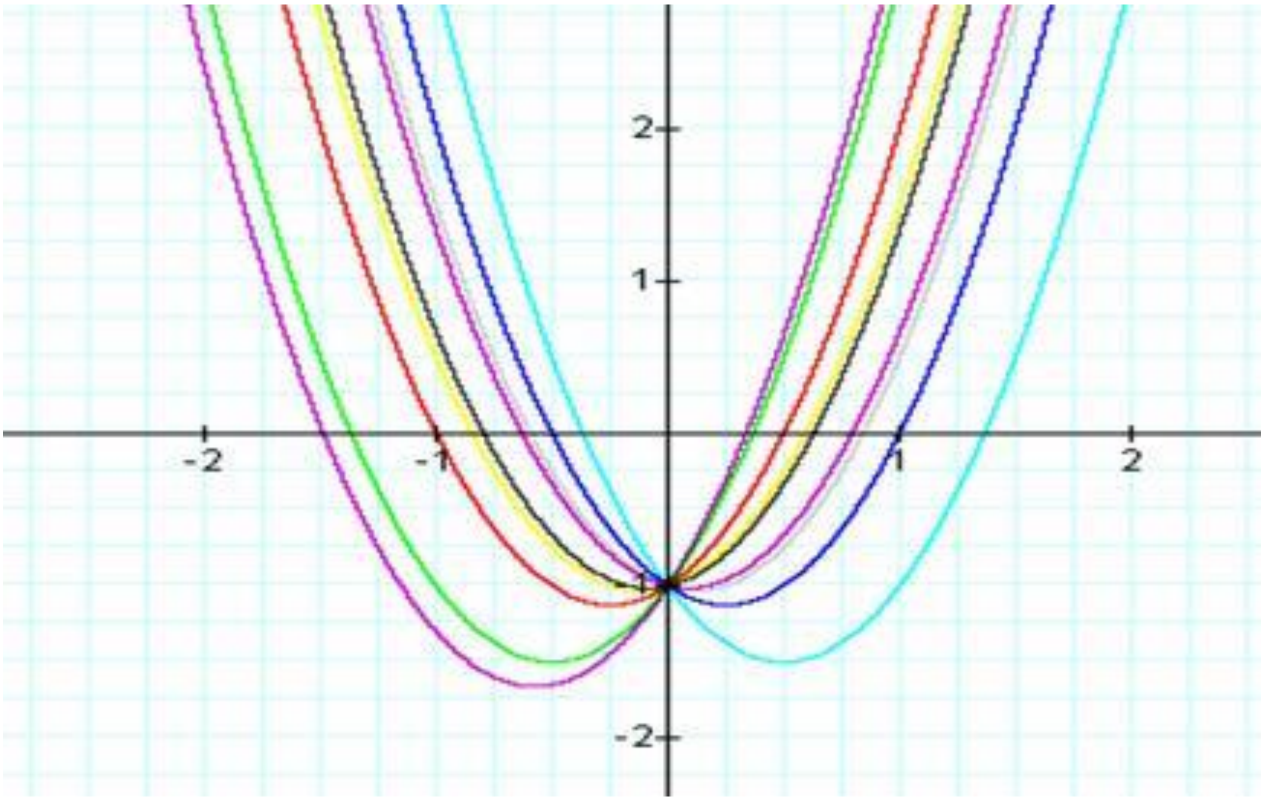

Fig.3

(c) c changes while **a** and **b** remain fixed **a≠0.**

If the coefficient c changes while a and b remain fixed, then the x-coordinate of the vertex $x = -b/2a$ remain fixed, but the y-coordinate

$$y = ax^2 + bx + c = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c = \frac{b^2}{4a} - \frac{b^2}{2a} + c =$$

$= -\frac{b^2}{4a} + c = -\frac{b^2-4ac}{4a} = -\frac{D}{4a}$ of the vertex changes. The graph of the parabola moves upwards when $c > 0$ and downwards when $c < 0$.

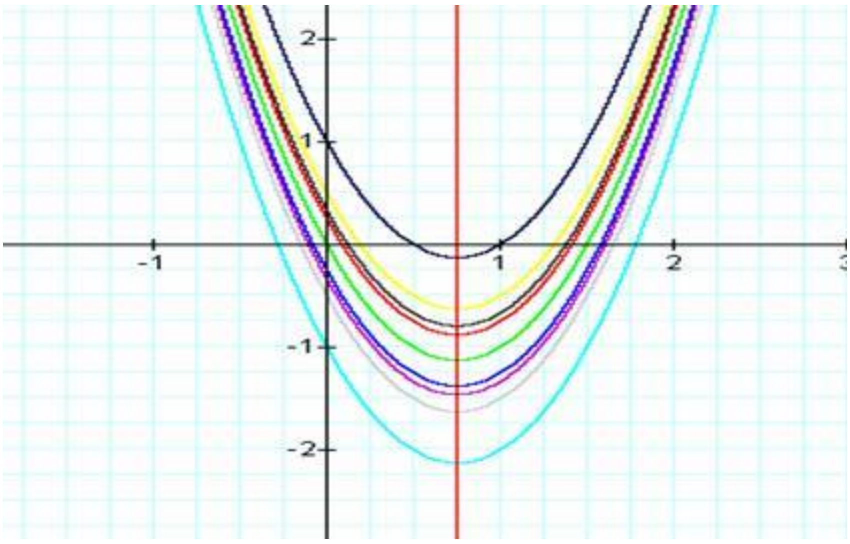

Fig.4