

### Answer on Question #53407 – Math – Algebra

$r(x) = p \cos^2(x) - 1$ ,  $p > 1$ . Sketch the graph of  $y=r(x)$  for  $-2\pi \leq x \leq 2\pi$  showing any points where the graph meets or crosses the coordinate axes.

#### Solution

$$y = r(x) = p \cos^2 x - 1$$

Let's sketch the graph for  $p > 1$  and find all the points, where the graph crosses the coordinate axes. Firstly, find these points:

crossing  $x$ -axis ( $y = 0$ ):

$$p \cos^2 x - 1 = 0$$

$$p \cos^2 x = 1$$

$$\cos^2 x = \frac{1}{p}$$

$$\cos x = \pm \sqrt{1/p}$$

$$x = 2\pi n \pm \cos^{-1}(\pm \sqrt{1/p}), n \in \mathbb{Z}$$

So, all the solutions:

$$x = 2\pi n + \cos^{-1}(\sqrt{1/p}), n \in \mathbb{Z}$$

$$x = 2\pi n + \cos^{-1}(-\sqrt{1/p}), n \in \mathbb{Z}$$

$$x = 2\pi n - \cos^{-1}(\sqrt{1/p}), n \in \mathbb{Z}$$

$$x = 2\pi n - \cos^{-1}(-\sqrt{1/p}), n \in \mathbb{Z}$$

For our interval the solutions are:

$$n = 0: x_1 = \cos^{-1}(\sqrt{1/p}), x_2 = \cos^{-1}(-\sqrt{1/p}), x_3 = -\cos^{-1}(\sqrt{1/p}), x_4 = -\cos^{-1}(-\sqrt{1/p})$$

$$n = 1: x_5 = 2\pi - \cos^{-1}(\sqrt{1/p}), x_6 = 2\pi - \cos^{-1}(-\sqrt{1/p})$$

$$n = -1: x_7 = \cos^{-1}(\sqrt{1/p}) - 2\pi, x_8 = \cos^{-1}(-\sqrt{1/p}) - 2\pi$$

crossing  $y$ -axis ( $x=0$ ):

$$y = p \cos^2 0 - 1 = p - 1$$

Then plot the graph and tick all these points:

