# Answer on Question #53404 – Math – Algebra

## Question

Solve the simultaneous equations

$$\begin{cases} 3y^{\frac{1}{3}} = 2x^{\frac{1}{2}} & (1) \\ 3y - 2x = 6 & (2) \end{cases}$$

## Solution

1. Take $x^{\frac{1}{2}}$ from (1)
   $x^{\frac{1}{2}} = \frac{3}{2}y^{\frac{1}{3}}$
   and substitute into (2):
   $x^{\frac{1}{2}} = \frac{3}{2}y^{\frac{1}{3}} \to\ 3y - 2\left(\frac{3}{2}y^{\frac{1}{3}}\right)^2 = 6 \to$
   $3y - \frac{9}{2}y^{\frac{2}{3}} = 6\ \to\ 2y - 3y^{\frac{2}{3}} = 4$
2. Introduce a new variable $t = y^{\frac{1}{3}}$
3. Rewrite equation in terms of this variable:
   $2y - 3y^{\frac{2}{3}} = 4\ \to$
   $$2t^3 - 3t^2 - 4 = 0 \quad (3)$$
4. Check if $\pm 1$ or $\pm 2$ are roots of equation (3):
   $$2 * 1^3 - 3 * 1^2 - 4 = -5 \neq 0$$
   $$2 * (-1)^3 - 3 * (-1)^2 - 4 = -9 \neq 0$$
   $$2 * 2^3 - 3 * 2^2 - 4 = 0$$
   $$2 * (-2)^3 - 3 * (-2)^2 - 4 = -32$$
   Thus, $t_1 = 2$ is a root of equation (3)
5. Divide $2t^3 - 3t^2 - 4$ by $t - 2$ in order to factor out $2t^3 - 3t^2 - 4$ :
   $\frac{2t^3 - 3t^2 - 4}{t-2} = 2t^2 + t + 2$
6. Solve obtained quadratic equation:
   $2t^2 + t + 2 = 0$
   $$D = 1 - 4 * 2 * 2 = -15$$
   $$t_2 = \frac{-1 + i\sqrt{15}}{4}; \quad t_3 = \frac{-1 - i\sqrt{15}}{4}$$
7. Back to $y$: $y = t^3$.
   $$y_1 = 8; \quad y_2 = \frac{11 - i3\sqrt{15}}{16}; \quad y_3 = \frac{11 + i3\sqrt{15}}{16}$$
8. Calculate respective $x$: $x = 3\left(\frac{y}{2} - 1\right)$
   $$x_1 = 9; \quad x_2 = \frac{-63 - i9\sqrt{15}}{32}; \quad x_3 = \frac{-63 + i9\sqrt{15}}{32}$$
9. Note that pairs $x_{2,3}, y_{2,3}$ have different phase in exponential representation that isn't fraction of $\pi$ (e.g. $y_2 \to e^{-itan^{-1}\left(\frac{3\sqrt{15}}{11}\right)}$ and $x_2 \to e^{i\left(tan^{-1}\left(\frac{\sqrt{15}}{7}\right) - \pi\right)}$). It means that those solutions do not satisfy (1) (they appear due to step 2).
10. The only one solution left $\to x = 9; y = 8$.

**Answer:** $x = 9; \;\; y = 8.$