Answer on Question #53403 - Math - Analytic Geometry

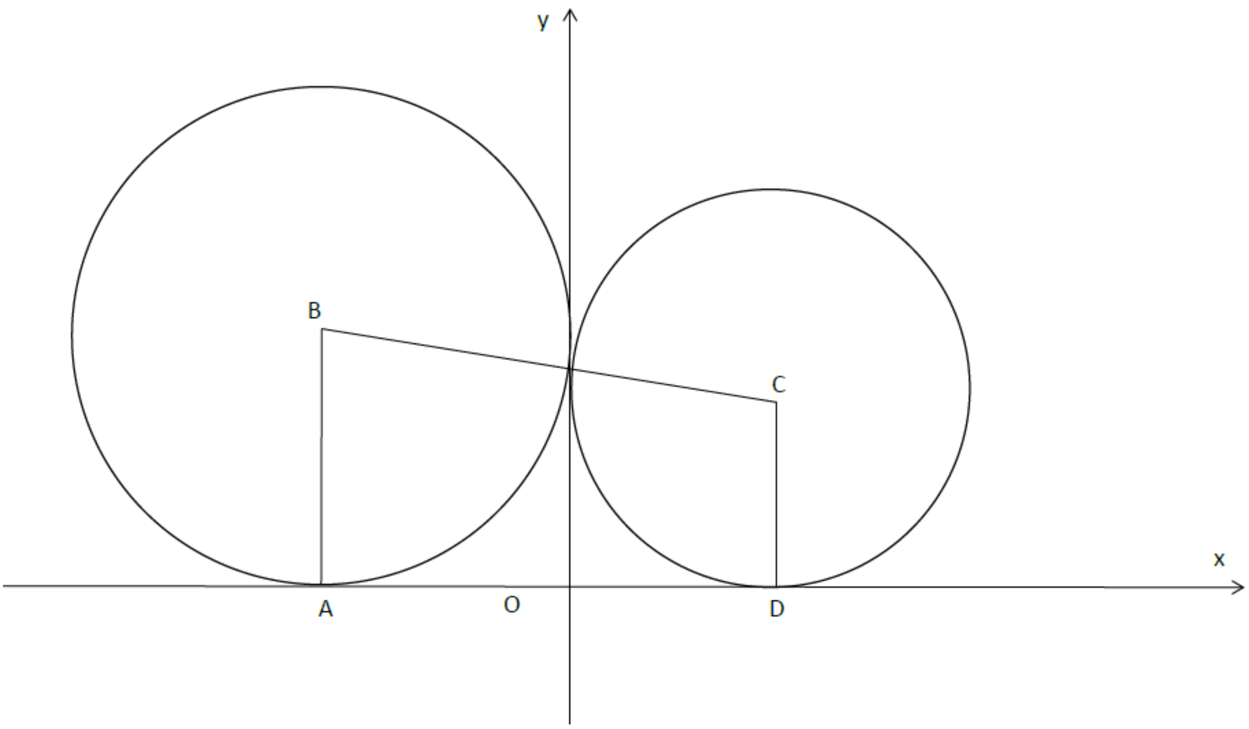
Two circles C1 and C2 both have the coordinate axes as tangents. Equation of C1 is $(x-a)^2 + (y-b)^2 = 25$ where $a < 0$ and $b > 0$ and equation of C2 is $(x-c)^2 + (y-d)^2 = 16$ where $c, d > 0$.

C1 touches the x axis at the point A and has its centre at the point B

C2 touches the x axis at the point D and has its centre at the point C

Find the area of the quadrilateral ABCD giving your answer as an exact fraction.

Solution



Based on the given information we drew the above figure. Thus the quadrilateral ABCD is trapezoid. Its area is given by

$$S = \frac{AB + CD}{2} AD$$

It's obvious that radii of C1 and C2 are

$$AB = \sqrt{25} = 5$$

$$CD = \sqrt{16} = 4$$

The line (AB) pathing through the centre of the circle and the line (y-axis) tangent to it are parallel, therefore the distance (AO) between these two lines is equal to the radius of the ircle C1. The same is correct for circle C2.

Since x-axis is tangent to C1 and C2, BA and CD are perpendicular to it as radii of these circles. Since y-axis is perpendicular to AD, then using the previous statement we obtain that BA is parallel to y-axis and CD is also parallel to y-axis. Since y-axis is tangent to both C1 and C2, then using previous statement we obtain that AO is equal to the radius of the circle C1 (AB) and DO is equal to the radius of the circle C2 (CD). Thus $AD = AO + DO = AB + CD$ and we obtain

$$S = \frac{AB + CD}{2} AD = \frac{(AB + CD)^2}{2} = \frac{(5 + 4)^2}{2} = \frac{81}{2}$$

Answer: $\frac{81}{2}$.