Answer on Question #53373 – Math – Trigonometry

Prove that

$$\frac{3\sin x+\sin(2x)}{1+3\cos x+\cos(2x)}=\tan x$$

where x is a constant.

Solution

We'll use next trigonometric identities

$$\sin(2x)=2\sin(x)\cos(x)\,;$$

$$\cos(2x)=\cos^2 x-\sin^2 x\,;$$

$$\sin^2 x=1-\cos^2 x.$$

Thus we have

$$\frac{3\sin x+\sin(2x)}{1+3\cos x+\cos(2x)}=\frac{3\sin x+2\sin(x)\cos(x)}{1+3\cos x+\cos^2 x-\sin^2 x}=$$

$$=\frac{\sin(x)\,(3+2\cos(x))}{1+3\cos x+\cos^2 x-(1-\cos^2 x)}=\frac{\sin(x)\,(3+2\cos(x))}{1+3\cos x+\cos^2 x-1+\cos^2 x}=$$

$$=\frac{\sin(x)\,(3+2\cos(x))}{3\cos x+2\cos^2 x}=\frac{\sin(x)\,(3+2\cos(x))}{\cos(x)\,(3+2\cos(x))}=\frac{\sin(x)}{\cos(x)}=\tan x.$$

So we proved

$$\frac{3\sin x+\sin(2x)}{1+3\cos x+\cos(2x)}=\tan x.$$