Answer on Question #53336 – Math – Algorithms | Quantitative Methods

Question Let $f(n) = 560n^3 + 3n + 107$ and $g(n) = 3n^3 + 5000n^2$. Which of the following is true?

a) $f(n)$ is $O(g(n))$, but $g(n)$ is not $O(f(n))$
b) $g(n)$ is $O(f(n))$, but $f(n)$ is not $O(g(n))$
c) $f(n)$ is not $O(g(n))$, and $g(n)$ is not $O(f(n))$
d) $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$.

Solution

In computer science "$f(n)$ is $O(g(n))$" means that $f(n)$ growth asymptotically no faster than $g(n)$. So there exist positive numbers k_1 and k_2 such that inequalities

$$k_1 g(n) \le f(n) \le k_2 g(n)$$

hold true.

Search bounds for $f(n)$ and $g(n)$:

$f(n) = 560n^3 + 3n + 107 \le 560n^3 + 3n^3 + 107 = 563n^3 + 107 \le 564n^3 + 107 =$
$= 188 \cdot 3n^3 + 107 \le 188 \cdot (3n^3 + 5000n^2) = 188 \cdot g(n)$, hence $\frac{1}{188} f(n) \le g(n)$;

$g(n) = 3n^3 + 5000n^2 \le 3n^3 + 5000n^3 = 5003n^3 \le 5040n^3 = 9 \cdot 560n^3 \le$
$\le 9(560n^3 + 3n + 107) = 9f(n)$, hence $\frac{1}{9} g(n) \le f(n)$.

We showed that

$$\frac{1}{9} g(n) \le f(n) \le 188 g(n)$$

and

$$\frac{1}{188} f(n) \le g(n) \le 9f(n)$$

We have polynomial expressions of the same order, hence answer is d) $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$.

Answer: d) $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$.