Answer on Question #53273 – Math – Analytic Geometry

Question:

A circle with area (25/9)*pi touches the x-axis at the point (4,0).

The point T is the furthest point on the circle from the origin O. Find the length of OT giving your answer as a simplified fraction.

Solution

If circle's area is (25/9)*pi, the formula for area of circle is S=pi*r^2, where r is the length of circle's radius, then

(25/9)*pi= pi*r^2, so r=5/3.

If circle's radius is 5/3 and circle touches the x-axis at point (4,0), then the equation of the circle is

$(x-4)^2+(y-5/3)^2$=25/9. The center of the circle is (4, 5/3).

furthest lie on a diameter

y = mx + c

gradient m = y/x = (5/3)/4=5/12

y = mx + c

5/3 = (5/12)*4 + c. so c=0.

The line from the center of the circle to the origin is y = 5x/12 . Find the intersections of line and circle.
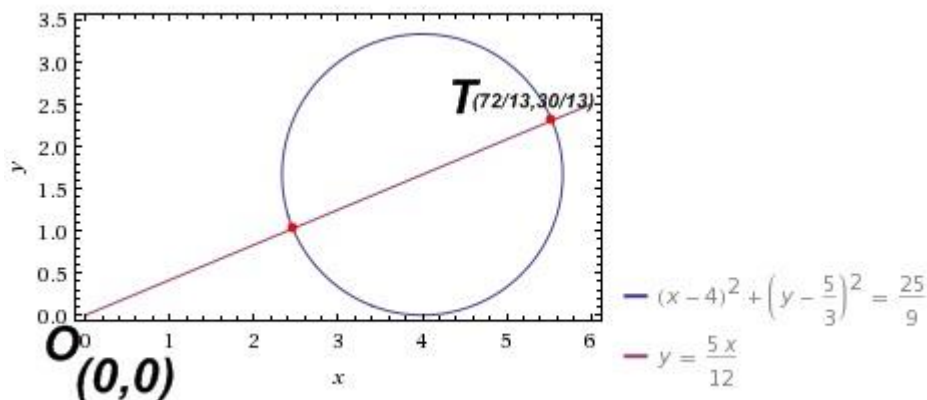
$(x-4)^2+((5x/12) -5/3)^2$=25/9

$x^2-8x+16+25x^2/144+25/9-25/18$=25/9

x=32/13; 72/13.

farthest from origin: (72/13, 30/13).

So the coordinates of T is (72/13, 30/13).

the length of OT is: OT=$((72/13)^2+(30/13)^2)^{1/2}$=6.

Answer: OT=6