Answer on Question #53230 – Math – Analytic Geometry

Show that the points A(2,-6,0),B(4,-9,6),C(5,0,2),D(7,-3,8) are concyclic.

Solution

In geometry, a set of points are said to be concyclic if they lie on a common circle. All concyclic points are at the same distance from the center of the circle.

$A(2,-6,0)$, $B=(4,-9,6)$, $C=(5,0,2)$, $D=(7,-3,8)$

There is a very easy way to determine whether four points lie on a circle if we know the distances between them. Conversely, if we know that four points lie on any circle, the Ptolemy's Theorem states that product of the diagonals of inscribed quadrilateral is equal to the sum of products of its opposite sides.

Let's find arrangement of the points (see Figure 1). $\overrightarrow{AB}=(2,-3,6)$, $|AB|=\sqrt{2^2+(-3)^2+6^2}=7$

$\overrightarrow{AC}=(3,6,2)$, $|AC|=\sqrt{3^2+6^2+2^2}=7$

$\overrightarrow{BD}=(3,6,2)$, $|BD|=\sqrt{3^2+6^2+2^2}=7$

$\overrightarrow{CD}=(2,-3,6)$, $|CD|=\sqrt{2^2+(-3)^2+6^2}=7$

$\overrightarrow{BC}=(1,9,-4)$, $|BC|=\sqrt{1^2+9^2+(-4)^2}=7\sqrt{2}$

$\overrightarrow{DA}=(-5,-3,-8)$, $|DA|=\sqrt{(-5)^2+(-3)^2+(-8)^2}=7\sqrt{2}$

Now let's test the statement of the Ptolemy's Theorem. Obvious that $|DA|$ and $|BC|$ are the diagonals, and $|AB|=|CD|=|AC|=|BD|$.

$$|AB|\cdot|CD|+|AC|\cdot|BD|=|BC|\cdot|DA|$$
$$7\cdot7+7\cdot7=7\sqrt{2}\cdot7\sqrt{2}$$
$$98=98$$

Thus, the points $A(2,-6,0)$, $B=(4,-9,6)$, $C=(5,0,2)$, $D=(7,-3,8)$ lie on a common circle and they are concyclic.

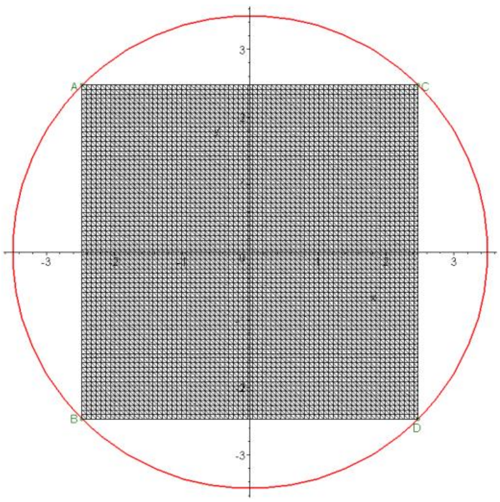

Figure 1.