Answer on Question #53148 – Math – Integral Calculus

Integrate $\int \frac{x^4}{(x-1)\sqrt{x+2}}dx$

Solution

$$\int \frac{x^4}{(x-1)\sqrt{x+2}}dx = \begin{vmatrix} \sqrt{x+2} = t & x^4 = (t^2-2)^4 \\ dx = 2tdt & x-1 = t^2-3 \end{vmatrix} = 2\int \frac{(t^2-2)^4}{t^2-3}dt =$$

$$= 2\int \frac{t^8 - 8t^6 + 24t^4 - 32t^2 + 16}{t^2-3}dt =$$

$$= 2\int \frac{t^6(t^2-3) + 5t^4(t^2-3) + 9t^2(t^2-3) - 5(t^2-3) + 1}{t^2-3}dt =$$

$$= 2\int \left(t^6 - 5t^4 + 9t^2 - 5 + \frac{1}{t^2-3}\right)dt = 2\left(\frac{t^7}{7} - t^5 + 3t^3 - 5t + \frac{1}{2\sqrt{3}}\log\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right|\right) + C =$$

$$= 2t\left(\frac{t^6}{7} - t^4 + 3t^2 - 5\right) + \frac{1}{\sqrt{3}}\log\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + C =$$

$$= 2\sqrt{x+2}\left(\frac{(x+2)^3}{7} - (x+2)^2 + 3(x+2) - 5\right) + \frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + C =$$

$= \frac{2}{7}\sqrt{x+2}(x^3 - x^2 + 5x - 13) + \frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + C,$

where C is an arbitrary real constant.

Answer: $\frac{2}{7}\sqrt{x+2}(x^3 - x^2 + 5x - 13) + \frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + C.$