Answer on Question #53134 – Math – Integral Calculus

Find the reduction formula of ∫sec^6(x)dx.

Solution

$$I_n = \int sec^n x dx = \int sec^{n-2} x sec^2 x dx = \int sec^{n-2} x d(tanx) =$$

$$= sec^{n-2} x tanx + (n-2) \int sec^{n-2} x tan^2 x dx =$$

$$= sec^{n-2} x tanx + (n-2) \int sec^{n-2} x (sec^2 x - 1) dx =$$

$$= sec^{n-2} x tanx + (n-2) I_{n-2} - (n-2) I_n \ \rightarrow$$

$$\rightarrow \ I_n = \frac{1}{n-1} sec^{n-2} x tanx + \frac{n-2}{n-1} I_{n-2}$$

Thus

$$I_6 = \frac{1}{5} sec^4\, xtanx + \frac{4}{5}\left(\frac{1}{3} sec^2 xtanx + \frac{2}{3} I_2\right) =$$

$$= \frac{1}{5} sec^4 xtanx + \frac{4}{15} sec^2 xtanx + \frac{8}{15} tanx + c,$$

where *c* is an arbitrary real constant.

So

$$\int sec^6 x dx = \frac{tanx}{15}(sec^4 x + 4sec^2 x + 8) + c,$$

where *c* is an arbitrary real constant.