Answer on Question #53133 – Math – Integral Calculus

Find the reduction formula of ∫cot^5(x)dx.

Solution

$$I_n = \int \cot^{\mathrm{n}} x\, dx = \int \cot^{\mathrm{n}-2} x \cot^2 x\, dx = \int \cot^{\mathrm{n}-2} x\, (\csc^2 x - 1) dx$$
$$= -\int \cot^{\mathrm{n}-2} x\, dx + \int \cot^{\mathrm{n}-2} x \csc^2 x\, dx.$$

$$\csc^2 x = -(\cot x)'.$$

$$\int \cot^{\mathrm{n}-2} x \csc^2 x\, dx = \int \cot^{\mathrm{n}-2} x\, d(\cot x) = -\frac{\cot^{\mathrm{n}-1} x}{n-1} + c,$$

where *c* is an arbitrary real constant.

The reduction formula of I_n:

$$I_n = -I_{n-2} - \frac{\cot^{\mathrm{n}-1} x}{n-1}.$$

The reduction formula of I_5:

$$I_5 = -I_3 - \frac{\cot^3 x}{3}.$$

$$I_3 = -I_1 - \cot x.$$

$$I_1 = \ln|\sin x| + c,$$

where *c* is an arbitrary real constant.

Thus

$$\int \cot^5 x\, dx = -(-\ln|\sin x| - \cot x) - \frac{\cot^3 x}{3} + c = \ln|\sin x| + \cot x - \frac{\cot^3 x}{3} + c,$$

where *c* is an arbitrary real constant.