Answer on Question #53132 – Math – Integral Calculus

Find the reduction formula of ∫tan6xdx

Solution

$$I_n = \int \tan^{\mathrm{n}} x\, dx = \int \tan^{\mathrm{n}-2} x \tan^2 x\, dx = \int \tan^{\mathrm{n}-2} x\, (\sec^2 x - 1) dx$$
$$= -\int \tan^{\mathrm{n}-2} x\, dx + \int \tan^{\mathrm{n}-2} x \sec^2 x\, dx.$$

$$\sec^2 x = (\tan x)'.$$

$$\int \tan^{\mathrm{n}-2} x \sec^2 x\, dx = \int \tan^{\mathrm{n}-2} x\, d(\tan x) = \frac{\tan^{\mathrm{n}-1} x}{n-1}.$$

The reduction formula of I_n:

$$I_n = -I_{n-2} + \frac{\tan^{\mathrm{n}-1} x}{n-1}.$$

The reduction formula of I_6:

$$I_6 = -I_4 + \frac{\tan^5 x}{5}.$$

$$I_4 = -I_2 + \frac{\tan^3 x}{3}.$$

$$I_2 = -I_0 + \tan x = -x + \tan x + C.$$

Thus

$\int \tan^6 x\, dx = -\left(-(-x + \tan x) + \frac{\tan^3 x}{3}\right) + \frac{\tan^5 x}{5} + C = -x + \frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \tan x + C$, where C is an arbitrary real constant.