Answer on Question #53131 – Math – Integral Calculus

Q. Find the reduction formula of ∫cos^12(x)dx.

Solution

$$I_n = \int cos^n x dx = \int cos^{n-1} x cosx dx = \int cos^{n-1} x d(sinx) =$$

$$= cos^{n-1} x sinx + (n-1) \int cos^{n-2} x sin^2 x dx =$$

$$= cos^{n-1} x sinx + (n-1) \int cos^{n-2} x(1 - cos^2) x dx =$$

$$= cos^{n-1} x sinx + (n-1) I_{n-2} + (n-1) I_n \rightarrow$$

$$\rightarrow \ I_n = \frac{1}{n} cos^{n-1} x sinx + \frac{n-1}{n} I_{n-2}$$

Thus,

$$I_{12} = \frac{1}{12} cos^{11} x sinx + \frac{11}{12} I_{10} = \frac{1}{12} cos^{11} x sinx + \frac{11}{12}\left(\frac{1}{10} cos^9 x sinx + \frac{9}{10} I_8\right) =$$

$$= \frac{1}{12} cos^{11} x sinx + \frac{11}{120} cos^9 x sinx + \frac{99}{120}\left(\frac{1}{8} cos^7 x sins + \frac{7}{8} I_6\right) =$$

$$= \frac{1}{12} cos^{11} x sinx + \frac{11}{120} cos^9 x sinx + \frac{99}{960} cos^7 x sinx + \frac{693}{960}\left(\frac{1}{6} cos^5 x sinx + \frac{5}{6} I_4\right) =$$

$$= \frac{1}{12} cos^{11} x sinx + \frac{11}{120} cos^9 x sinx + \frac{33}{320} cos^7 x sinx + \frac{231}{1920} cos^5 x sinx +$$

$$+ \frac{1155}{1920}\left(\frac{1}{4} cos^3 x sinx + \frac{3}{4} I_2\right) =$$

$$= \frac{1}{12} cos^{11} x sinx + \frac{11}{120} cos^9 x sinx + \frac{33}{320} cos^7 x sinx + \frac{231}{1920} cos^5 x sinx +$$

$$+ \frac{1155}{7680} cos^3 x sinx + \frac{3465}{7680}\left(\frac{1}{2} cosx sinx + \frac{1}{2} I_0\right)$$

But $I_0 = \int dx = x + c$

So

$$\int \cos^{12} x dx =$$

$$= \frac{\sin 2x}{30720}\left(1280\cos^{10}x + 1408\cos^{8}x + 1584\cos^{6} + 1848\cos^{4}x + 2310\cos^{2}x + 3465\right) +$$

$$+ \frac{3465}{15360}x + c,$$

where *c* is an arbitrary real constant.