Answer on Question #53128 – Math – Integral Calculus

Find the reduction formula of ∫sin8xdx.

Solution

$$I_n = \int sin^n x dx = \int sin^{n-1} x sinx dx = -\int sin^{n-1} x d(cosx) =$$

$$= -sin^{n-1} x cosx + (n-1) \int sin^{n-2} x cos^2 x dx =$$

$$= -sin^{n-1} x cosx + (n-1) \int sin^{n-2} x (1 - sin^2 x) dx =$$

$$= -sin^{n-2} x cosx + (n-1) I_{n-2} - (n-1) I_n \rightarrow$$

$$\rightarrow \quad I_n = -\frac{1}{n} sin^{n-1} x cosx + \frac{n-1}{n} I_{n-2}$$

Thus

$$I_8 = -\frac{1}{8} sin^7 x cosx + \frac{7}{8} I_6 = -\frac{1}{8} sin^7 x cosx + \frac{7}{8}\left(-\frac{1}{6} sin^5 x cosx + \frac{5}{6} I_4\right) =$$

$$= -\frac{1}{8} sin^7 x cosx - \frac{7}{48} sin^5 x cosx + \frac{35}{48}\left(-\frac{1}{4} sin^3 x cosx + \frac{3}{4} I_2\right) =$$

$$= -\frac{1}{8} sin^7 x cosx - \frac{7}{48} sin^5 x cosx - \frac{35}{192} sin^3 x cosx + \frac{105}{192}\left(-\frac{1}{2} sinx cosx + \frac{1}{2} I_0\right) =$$

$$= -\frac{1}{8} sin^7 x cosx - \frac{7}{48} sin^5 x cosx - \frac{35}{192} sin^3 x cosx - \frac{105}{384} sinx cosx + \frac{105}{384} I_0$$

But $I_0 = \int dx = x + c$**, where** c **is an arbitrary real constant.**

So

$$\int sin^8 x dx = -\frac{sinx cosx}{384}(48 sin^6 x + 56 sin^4 x + 70 sin^2 x + 105) + \frac{105}{384} x + c$$