Answer on Question #53074 – Math – Analytic Geometry

Question

I need to clear up something. What's the difference between the midpoint of two coordinates and finding half the distance between the points? The formula for midpoint is ((x2 + x1)/2, (y2 + y1)/2). The formula for distance between two points is √((x2 - x1)^2 + (y2 - y1)^2)?

Solution

To understand the difference let`s use the geometric interpretation and consider the following problem. Assume that two points $A=\{x_1;\ y_1\}$ and $B=\{x_2;\ y_2\}$ are given, and let AB be the line segment connecting the given points. Find the length of this segment and coordinates of its midpoint C=$\{x, y\}$.

Let`s sketch out the figure concerning our problem (see fig.1) and introduce the following notation: the point $O = \{x_0;y_0\}$ is the origin of coordinates; $\overrightarrow{OA} = \vec{r}_1$, $\overrightarrow{OB} = \vec{r}_2$, $\overrightarrow{OC} = \vec{r}$, where $\vec{r}_1, \vec{r}_2, \vec{r}$ are the radius vectors from the point O to the points A, B and C respectively; $\overrightarrow{AB}$ is vector from given point A to the point B.

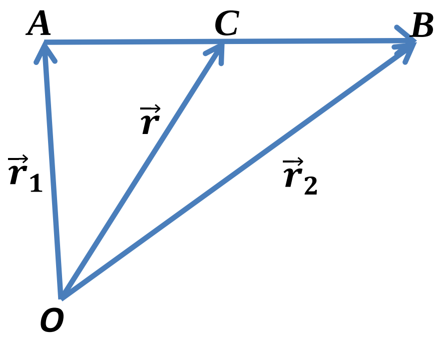

Fig.1

Using the rule of vector addition we can write (see fig.1)

$$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB},$$

$$\vec{r}_1 + \overrightarrow{AC} = \vec{r},\ \ \vec{r} + \overrightarrow{CB} = \vec{r}_{2.} \qquad (1)$$

As C is midpoint of AB, then $\overrightarrow{AC} = \overrightarrow{CB}$ and from (1) we get

$$\vec{r} - \vec{r}_1 = \vec{r}_{2.} - \vec{r}. \qquad (2)$$

or

$$\vec{r} = \frac{\vec{r}_1 + \vec{r}_2}{2}. \qquad (3)$$

By the definition of vector in a coordinate representation

$$\vec{r} = \{x - x_0; y - y_0\},\ \vec{r}_1 = \{x_1 - x_0; y_1 - y_0\}, \qquad \vec{r}_2 = \{x_2 - x_0; y_2 - y_0\}. \tag{4}$$

Putting $x_0 = 0, y_0 = 0$ we have

$$\vec{r} = \{x; y\},\ \vec{r}_1 = \{x_1; y_1\}, \qquad \vec{r}_2 = \{x_2; y_2\}. \tag{4a}$$

Thus, the relation (3) in a coordinate form is

$$\vec{r} = \{x; y\},\ x = \frac{x_1+x_2}{2},\ y = \frac{y_1+y_2}{2}. \tag{5}$$

Now we can write for the midpoint *C*:

$$C = \{x, y\} = \left\{\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right\}. \tag{6}$$

According to definition, the distance between the two points *A* and *B* is the length *d* of segment *AB* connecting these points.

Let`s write the relations for the vector $\overrightarrow{AB}$ (see fig.1):

$$\overrightarrow{AB} = \vec{r}_{2.} - \vec{r}_1 \Longrightarrow \overrightarrow{AB} = \{x_2 - x_1; y_1 - y_2\}. \tag{7}$$

In our case the length *d* of segment *AB* is equal to the length of vector $\overrightarrow{AB}$:

$$d = \left|\overrightarrow{AB}\right| = \sqrt{\left(\overrightarrow{AB} \cdot \overrightarrow{AB}\right)} = \sqrt{(x_2 - x_1)^2 + (y - y_1)^2}. \tag{8}$$

Therefore, the formula (8) defines the length of segment (or the distance between the two points), whereas the formula (6) defines the coordinates of midpoint that lies on the line connecting the two given points.