Answer on Question #53015 – Math – Analytic Geometry

Find the value of k so that the line $y + 5 = k(x - 3)$ is numerically 3 unit distant from the origin.

Solution

$y = k(x-3) - 5$

Let's write the equation of a line in the normal form:

$$Ax + By = C,$$
$$Ax + By - C = 0$$
$$kx - y - 3k - 5 = 0$$
$$A = k, B = -1, C = 3k + 5$$

Let's normalize the equation of the line dividing it by the square root of $\sqrt{A^2+B^2}$ or multiplying by $\frac{1}{\sqrt{A^2+B^2}}$.

$$\frac{A}{\sqrt{A^2+B^2}}x + \frac{B}{\sqrt{A^2+B^2}}y - \frac{C}{\sqrt{A^2+B^2}} = 0$$

$$\sqrt{A^2+B^2} = \sqrt{k^2+1}$$

$$\frac{k}{\sqrt{k^2+1}}x + \frac{-1}{\sqrt{k^2+1}}y - \frac{3k+5}{\sqrt{k^2+1}} = 0$$

This normalized equation contains very important information: the coordinates of the unit normal vector $\vec{n} = (\frac{k}{\sqrt{k^2+1}}, \frac{-1}{\sqrt{k^2+1}})$ and the distance from the origin $d = \left|\frac{3k+5}{\sqrt{k^2+1}}\right|$:

According to the statement of the problem: $\left|\frac{3k+5}{\sqrt{k^2+1}}\right| = 3$.

Let's solve the equation and find the value (or values) of k .

$$\frac{3k+5}{\sqrt{k^2+1}} = \pm 3$$
$$\frac{(3k+5)^2}{k^2+1} = 9 \Rightarrow \frac{9k^2+30k+25-9k^2-9}{k^2+1} = 0$$
$$\frac{30k+16}{k^2+1} = 0 \Rightarrow 30k+16 = 0 \Rightarrow k = -\frac{8}{15}.$$

So, the equation of the line $y + 5 = k(x - 3)$ which is numerically 3 unit distant from the origin is

$$y + 5 = -\frac{8}{15}(x-3)$$
.

Now, let's check the answer: $d = \left|\dfrac{3\cdot(-\frac{8}{15})+5}{\sqrt{\frac{64}{225}+1}}\right| = \left|\dfrac{-\frac{8}{5}+5}{\sqrt{\frac{289}{225}}}\right| = \left|\dfrac{\frac{17}{5}}{\frac{17}{15}}\right| = 3$.

Answer: $k = -\frac{8}{15}$ **.**