Answer on Question #52898 – Math – Trigonometry

$\sin 2\theta = \cos 3\theta$

Solution

Method 1

Since $\sin 2\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$, cosine is a 2π-periodic, even function, then we obtain

$$\sin 2\theta = \cos 3\theta \Rightarrow \cos\left(\frac{\pi}{2} - 2\theta\right) = \cos 3\theta \Rightarrow \left[\begin{array}{l} \frac{\pi}{2} - 2\theta = 3\theta - 2\pi k, k \in Z \\ \frac{\pi}{2} - 2\theta = -3\theta + 2\pi k, k \in Z \end{array}\right. \Rightarrow \left[\begin{array}{l} \theta = \frac{1+4k}{10}\pi, k \in Z \\ \theta = \left(2k - \frac{1}{2}\right)\pi, k \in Z \end{array}\right.$$

Thus we found two collections of solutions

Answer: $\left[\begin{array}{l} \theta = \frac{1+4k}{10}\pi, k \in Z \\ \theta = \left(2k - \frac{1}{2}\right)\pi, k \in Z \end{array}\right.$

Method 2

Given $\sin 2\theta = \cos 3\theta$

Taking into account formula $\sin 2\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$, obtain that

$$\cos\left(\frac{\pi}{2} - 2\theta\right) = \cos 3\theta$$

Apply the formula $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$ to the previous equality and come to

$$\cos\left(\frac{\pi}{2} - 2\theta\right) - \cos 3\theta = 0$$

$$-2\sin\left(\frac{\pi/2 - 2\theta + 3\theta}{2}\right)\sin\left(\frac{\pi/2 - 2\theta - 3\theta}{2}\right) = 0$$

$\sin\left(\frac{\pi/2 - 2\theta + 3\theta}{2}\right) = 0$ or $\sin\left(\frac{\pi/2 - 2\theta - 3\theta}{2}\right) = 0$

Recall that $\sin(x) = 0$ gives $x = k\pi$, k is integer. Thus,

$\frac{\pi}{4} + \frac{\theta}{2} = k\pi$, that is, $\theta = \left(2k - \frac{1}{2}\right)\pi$, or

$\frac{\pi}{4} - \frac{10\theta}{4} = k\pi$, denote $k = -l$, therefore $\frac{\pi}{4} - \frac{10\theta}{4} = -l\pi$, hence $\theta = \frac{4l+1}{10}$.

Answer: $\left[\begin{array}{l} \theta = \frac{1+4k}{10}\pi, k \in Z \\ \theta = \left(2k - \frac{1}{2}\right)\pi, k \in Z \end{array}\right.$