

## Answer on Question #52829 - Math – Analytic Geometry

Show that the point A(2,-6,0), B(4,-9,6), C(5,0,2), D(7,-3,8) are concyclic.

### Solution

First of all, for A, B, C and D to be concyclic they all should lie in a single plane. It is true if the following condition holds:

$$\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 0$$

Since  $\overrightarrow{AB} = (4, -9, 6) - (2, -6, 0) = (2, -3, 6)$ ,  $\overrightarrow{AC} = (5, 0, 2) - (2, -6, 0) = (3, 6, 2)$  and

$\overrightarrow{AD} = (7, -3, 8) - (2, -6, 0) = (5, 3, 8)$ , we obtain

$$\begin{aligned} \overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) &= \begin{vmatrix} 2 & -3 & 6 \\ 3 & 6 & 2 \\ 5 & 3 & 8 \end{vmatrix} = 2 \begin{vmatrix} 6 & 2 \\ 3 & 8 \end{vmatrix} + 3 \begin{vmatrix} 3 & 2 \\ 5 & 8 \end{vmatrix} + 6 \begin{vmatrix} 3 & 6 \\ 5 & 3 \end{vmatrix} = 2 \cdot 42 + 3 \cdot 14 - 6 \cdot 21 = \\ &= 42 \neq 0 \end{aligned}$$

It means that these 4 points don't lie in a single plane and therefore they can't be concyclic.

**Answer:** not concyclic.