Answer on Question #52828 – Math – Analytic Geometry

Find the equation of cone with vertex (alpha,beta,gama)and base y2-4ax=0,z=0.

Solution

Any line through vertex (α,β,γ) is $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$.

Any point on this line is $(x,y,z)=(lr+\alpha, mr+\beta, nr+\gamma)$.

It will lie on the given parabola $y^2-4ax=0$, $z=0$ if, for the same value of r, we have

$$\begin{cases} nr+\gamma=0 \\ (mr+\beta)^2-4\cdot a(lr+\alpha)=0 \end{cases}$$

Then

$$\begin{cases} r=-\gamma/n \\ (mr+\beta)^2-4\cdot a(lr+\alpha)=0 \end{cases} \Rightarrow \begin{cases} r=-\gamma/n \\ \left(\beta-\gamma\frac{m}{n}\right)^2-4\cdot a\left(\alpha-\gamma\frac{l}{n}\right)=0 \end{cases}$$

So, eliminating l, m, n

$$\left(\beta-\gamma\frac{m}{n}\right)^2-4\cdot a\left(\alpha-\gamma\frac{l}{n}\right)=0 \Rightarrow (\beta n-\gamma m)^2-4\cdot an(\alpha n-\gamma l)=0$$

$$\left(\beta(z-\gamma)-\gamma(y-\beta)\right)^2-4\cdot a\alpha(z-\gamma)^2+4a\gamma(z-\gamma)(x-\alpha)=0$$

The equation of cone is $z^2\left(\beta^2-4a\alpha\right)+4a\gamma xz+4a\alpha\gamma z-2\beta\gamma yz-4a\gamma^2x+\gamma^2y^2=0$.