Answer on Question #52768 – Math – Analytic Geometry

Find the line of intersection of the two planes x+3y+z=5 and 4x+y+2z=15

Solution

To obtain the parametric equations of the line of intersection, we need: (1) to find a directional vector of this line and (2) to find a certain point on this line.

(1) This line lies on the first plane; therefore, it is orthogonal to the normal vector $\overline{n_1}$. This line lies on the second plane; therefore, it is orthogonal to the normal vector $\overline{n_2}$. Their cross product $\overline{n_1} \times \overline{n_2}$. is also orthogonal to $\overline{n_1}$ and $\overline{n_2}$; therefore, we can take this vector as $\bar{v}$, the directional vector of this line. We need to calculate this cross product. it is

$$\overline{n_1} \times \overline{n_2} =< 1,3,1 >\times< 4,1,2 >=< 3\cdot 2-1\cdot 1, 1\cdot 4-2\cdot 1, 1\cdot 1-3\cdot 4 >=< 5,2,-11 >.$$

(2) We have a system of equations:

$$\begin{cases} x+3y+z=5 \\ 4x+y+2z=15 \end{cases}$$

This system contains three variables and two equations. Such systems generally have infinitely many solutions. This one is not an exception. But we need to find only one point (doesn't matter which one). So set z = 0, then we have:

$$\begin{cases} x+3y=5 \\ 4x+y=15 \end{cases}$$

Therefore, $x = 5 - 3y$, $4(5-3y)+y=15$, $20-11y=15$, $11y=5$, $y=\frac{5}{11}$, $x=\frac{40}{11}$. The point $\left(\frac{40}{11}, \frac{5}{11}, 0\right)$ lies on this line. And we immediately obtain the parametric equations of the line:

$$x = \frac{40}{11} + 5t, \qquad y = \frac{5}{11} + 2t, \qquad z = -11t.$$