Answer on Question #52767 – Math – Analytic Geometry

Question

Find the equation of the plane through the following three points:

(1,2,3) (2,4,5) and (4,5,7)

Solution

A(1,2,3), B(2,4,5) and C(4,5,7). Let X(*x*, *y*, *z*).

$\overrightarrow{AB}$ = <2-1, 4-2, 5-3> = <1, 2, 2>

$\overrightarrow{AC}$ = <4-1, 5-2, 7-3> = <3, 3, 4>

$\overrightarrow{AX}$ = <*x*-1, *y*-2, *z*-3>

Method 1

The cross product (or vector product) of $\overrightarrow{AC}$ and $\overrightarrow{AB}$ is given by

$$\overrightarrow{AC}\times\overrightarrow{AB}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 3 & 4\\ 1 & 2 & 2\end{vmatrix}=\vec{i}\begin{vmatrix}3 & 4\\ 2 & 2\end{vmatrix}-\vec{j}\begin{vmatrix}3 & 4\\ 1 & 2\end{vmatrix}+\vec{k}\begin{vmatrix}3 & 3\\ 1 & 2\end{vmatrix}=(3\cdot 2-2\cdot 4)\vec{i}-(3\cdot 2-1\cdot 4)\vec{j}+$$
$$+(3\cdot 2-1\cdot 3)\vec{k}=-2\vec{i}-2\vec{j}+3\vec{k}.$$

Since $\overrightarrow{AX}\perp\left(\overrightarrow{AC}\times\overrightarrow{AB}\right)$, therefore the dot product (or scalar product) of $\overrightarrow{AX}$ and $\overrightarrow{AC}\times\overrightarrow{AB}$ is zero.

Thus,

$$<x-1,y-2,z-3>\cdot<-2,-2,3>=\ -2(x-1)-2(y-2)+3(z-3)=0$$

$$-2x+2-2y+4+3z-9=0$$

$$-2x-2y+3z-3=0$$

Multiply by (-1) and obtain

$$2x+2y-3z+3=0$$

Method 2

Vectors $\overrightarrow{AX},\overrightarrow{AB},\ \overrightarrow{AC}$ lie in one plane, therefore their scalar triple product is zero, therefore

$$\begin{vmatrix}x-1 & y-2 & z-3\\ 1 & 2 & 2\\ 3 & 3 & 4\end{vmatrix}=(x-1)\begin{vmatrix}2 & 2\\ 3 & 4\end{vmatrix}-(y-2)\begin{vmatrix}1 & 2\\ 3 & 4\end{vmatrix}+(z-3)\begin{vmatrix}1 & 2\\ 3 & 3\end{vmatrix}=$$
$$=(x-1)(2\cdot 4-3\cdot 2)-(y-2)(1\cdot 4-3\cdot 2)+(z-3)(1\cdot 3-3\cdot 2)$$
$$=2(x-1)+2(y-2)-3(z-3)=2x+2y-3z+(-2-4+9)$$
$$=2x+2y-3z+3=0$$