Answer on Question #52765 – Math – Integral Calculus

Question

Calculate the integral of the following functions:

a) sin^2 (x)

b) 1/(√x^2 -1)

Solution

a) $\int \sin^2(x)dx = \int \frac{1-\cos(2x)}{2}dx = = \frac{x}{2} - \frac{1}{4} \cdot \sin(2x) + C = \frac{1}{2} \cdot (x - \frac{\sin(2x)}{2}) + C =$

$= \frac{1}{2} \cdot (x - \sin(x)\cos(x)) + C,$

where C is an arbitrary real constant;

b) $\int \frac{1}{(\sqrt{x^2} - 1)} dx = \int \frac{dx}{|x| - 1} = \begin{cases} \ln|x-1| + C, \text{if } x > 0 \\ -\ln|x+1| + C, if\ x < 0 \end{cases}$

To calculate integral of $1/\sqrt{x^2-1}$,

substitute $x = cosht$,

$dx = (cosht)'_t dt = (sinht)dt,$

apply $cosh^2t - sinh^2t = 1,$

$cosh^2t - 1 = sinh^2t,$

$x^2 - 1 = sinh^2t,$

$$\frac{dx}{\sqrt{x^2-1}} = \frac{(\sinh t)dt}{\sqrt{cosh^2t - 1}} = \frac{\sinh t\, dt}{\sqrt{sinh^2t}} = dt$$

Rewrite

$x = cosht,$

$x = \frac{e^t + e^{-t}}{2},$

$e^t + \frac{1}{e^t} = 2x,$

$e^{2t} - (2x)e^t + 1 = 0,$

$e^t = \frac{2x + \sqrt{4x^2-4}}{2} = x + \sqrt{x^2-1},$

$t = ln\left|x + \sqrt{x^2-1}\right|.$

Finally

$\int \frac{dx}{\sqrt{x^2-1}} = \int dt = t + C = ln\left|x + \sqrt{x^2-1}\right| + C,$

where C is an arbitrary real constant.

Answer:

a) $\frac{1}{2} \cdot (x - \sin(x)\cos(x)) + C$

b) log(x – 1) + C