

Answer on Question #52641 – Math – Trigonometry

Question:

What is the actual value of $\arcsin(-15/17) + \arcsin(-30/36)$. Use $\arcsin x + \arcsin y$ formula.

Solution:

$$\arcsin x + \arcsin y =$$

$$= \begin{cases} \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{if } xy \leq 0 \text{ or } x^2 + y^2 \leq 1, \\ \pi - \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{if } x > 0, y > 0 \text{ and } x^2 + y^2 > 1, \\ -\pi - \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{if } x < 0, y < 0 \text{ and } x^2 + y^2 > 1. \end{cases}$$

Because $x = -\frac{15}{17} < 0$, $y = -\frac{30}{36} < 0$ and

$$x^2 + y^2 = \left(-\frac{15}{17}\right)^2 + \left(-\frac{30}{36}\right)^2 = \frac{15325}{10404} \approx 1.473 > 1, \text{ then we apply}$$

$$\begin{aligned} \arcsin x + \arcsin y &= -\pi - \arcsin\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) = \\ &= -\pi - \arcsin\left(\left(-\frac{15}{17}\right)\sqrt{1-\left(-\frac{30}{36}\right)^2} + \left(-\frac{30}{36}\right)\sqrt{1-\left(-\frac{15}{17}\right)^2}\right) = \\ &= -\pi - \arcsin\left(-\frac{240 + 90\sqrt{11}}{17 * 36}\right) = -\pi - 1.075643, \text{ that is,} \\ &\text{approximately } -\mathbf{241.63}^\circ. \end{aligned}$$