Answer on Question #52565 – Math – Trigonometry

If $arc\ sin\ x\ + arc\ sin\ y\ + arc\ sin\ z\ = \pi$. Prove that $[x\sqrt{1-x^2}]\ + [y\sqrt{1-y^2}] + [z\ \sqrt{1-z^2}] = 2xyz$.
Show the process please

Solution

Let $arc\ sin\ x = A, arc\ sin\ y = B, arc\ sin\ z = C$. So,

$$A + B + C = \pi.$$

Then

$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C.$$

Here is proof of it:

$$\begin{aligned}\sin 2A + \sin 2B + \sin 2C &= 2\sin(A+B)\cos(A-B) + 2\sin C\cos C \\ &= 2\sin(\pi - C)\cos(A-B) + 2\sin C\cos C = 2\sin(C)\cos(A-B) + 2\sin C\cos C \\ &= 2\sin(C)\left(\cos(A-B) + \cos\big(\pi - (A+B)\big)\right) = 2\sin(C)\left(\cos(A-B) - \cos(A+B)\right) \\ &= 2\sin(C)\left(2\sin B\sin A\right) = 4\sin A\sin B\sin C.\end{aligned}$$

$$2\sin A\cos A + 2\sin B\cos B + 2\sin C\cos C = 4\sin A\sin B\sin C$$

or

$$\sin A\cos A + \sin B\cos B + \sin C\cos C = 2\sin A\sin B\sin C.$$

We know that

$$\begin{aligned}\sin A\cos A + \sin B\cos B + \sin C\cos C &= \sin A\sqrt{1-\sin^2 A} + \sin B\sqrt{1-\sin^2 B} + \sin C\sqrt{1-\sin^2 C} = \\ &= \left[x\sqrt{1-x^2}\right] + \left[y\sqrt{1-y^2}\right] + \left[z\sqrt{1-z^2}\right],\end{aligned}$$

$$2\sin A\sin B\sin C = 2xyz.$$

Thus,

$$\left[x\sqrt{1-x^2}\right] + \left[y\sqrt{1-y^2}\right] + \left[z\sqrt{1-z^2}\right] = 2xyz.$$