Answer on Question #52550 – Math – Trigonometry

What is the actual value of arc sin(12/13) +arc sin(4/5)+arc sin(1). Show the process using arc sinx +arc sin y formula?

Using formula for the first two it comes pi - arc sin (56/65) then + arc sin 1 which is pi/2. So the final result is 3pi/2 - arc sin (56/65) . And if i don't write pi/2 instead of arc sin 1, then the result is pi - arc sin (56/65) +arc sin 1 = pi + arc sin (33/65) using formula for arc sin 1 - arc sin (56/65). i m lil bit confused which one is correct ? is there any use of principal value

The calculator shows its approximately 210 degree , means its a third quadrent angle .

Normally we know arc sinx +arc sin y = arc sin [x*sqrt(1-y^2) +y*sqrt(1-x^2)]. But if we use this , the answer comes arc sin (56/65) + pi/2 , which does not match with the calculator result . So is there any regulation for arc sinx +arc sin y formula and is it applicable for (arc sinx +arc sin y) < pi/2 ?????

Solution

$$\sin^{-1}x+\sin^{-1}y=\begin{cases}\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right);\ x^2+y^2\le 1\ or\ x^2+y^2>1, xy<0\\ \pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right); x^2+y^2>1, 0<x,y\le 1\\ -\pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right); x^2+y^2>1, -1<x,y\le 0\end{cases}$$

Thus,

$$\sin^{-1}\frac{12}{13}+\sin^{-1}\frac{4}{5}=\left|\left(\frac{12}{13}\right)^2+\left(\frac{4}{5}\right)^2>1, 0<\frac{12}{13},\frac{4}{5}\le 1\right|=\pi-\sin^{-1}\left(\frac{4}{5}\sqrt{1-\left(\frac{12}{13}\right)^2}+\frac{12}{13}\sqrt{1-\left(\frac{4}{5}\right)^2}\right)$$

$$=\pi-\sin^{-1}\left(\frac{4}{13}+\frac{36}{65}\right)=\pi-\sin^{-1}\left(\frac{56}{65}\right).$$

Then

$$\pi-\sin^{-1}\left(\frac{56}{65}\right)+\sin^{-1}1=\frac{3\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)\approx 210°.$$

But it really equals

$$\pi+\sin^{-1}\left(\frac{36}{65}\right)\approx 210°.$$

So, you saw just trigonometric identity, not a mistake!