Question

A survey was conducted to measure the number of hours per week adults in the United States spend on home computers. In the survey, the number of hours was normally distributed, with a mean of 7 hours and a standard deviation of 1 hour. Survey participants are randomly selected.

a) Find the probability that the hours spent on the home computer by the participant are less than 4.5 hours per week,

b) find the probability that the hours spent on the home computer by the participant are more than 9.5 hours per week.

Solution

a) By X denote the number of hours per week adults in the United States spend on home computers. $X \sim N(7,1)$

$$P(X < 4.5) = P(X - 7 < 4.5 - 7) = P(X - 7 < -2.5) = P(\zeta < -2.5),$$

where $\zeta \sim N(0,1)$.

So,

$$P(\zeta < -2.5) = \Phi(-2.5) = \int_{-\infty}^{-2.5} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 0.00620967$$

b)

Method 1

$$P(X > 9.5) = P(X - 7 > 9.5 - 7) = P(X - 7 > 2.5) = P(\zeta > 2.5) =$$

$$= 1 - P(\zeta < 2.5) = 1\text{-}0.99379033 = 0.00620967$$

Method 2

Normal distribution is symmetric around the mean, which is 7, but 4.5-7= - 2.5, 9.5-7=2.5, it means that $|4.5 - 7| = |9.5 - 7| = 2.5$. In other words points 4.5 and 9.5 are symmetric around the mean. It implies *P(X>9.5)= P(X<4.5)=0.0620967* as calculated in the previous question.