Answer on Question #52103 – Math – Integral Calculus

Integrate with respect to x: $\int_{-1}^{3} \frac{x}{7+x^2} dx$

Solution

$$\int_{-1}^{3} \frac{x}{7+x^2} dx = \frac{1}{2}\int_{-1}^{3} \frac{1}{7+x^2} d(x^2+7) = \left| substitution\ t = x^2+7, \qquad \frac{dt}{t} = d(\ln(t)) \right|$$
$$= \frac{1}{2}\ln(x^2+7)\Big|_{-1}^{3} = \frac{1}{2}\left(\ln(3^2+7) - \ln((-1)^2+7)\right) = \frac{1}{2}\ln\left(\frac{3^2+7}{(-1)^2+7}\right) = \frac{\ln 2}{2}.$$

Answer: $\frac{\ln 2}{2}$.

$$\int_{1}^{4} \left(x + \frac{1}{\sqrt{x}}\right) dx$$

Solution

$$\int_{1}^{4} \left(x + \frac{1}{\sqrt{x}}\right) dx = \int_{1}^{4} x dx + \int_{1}^{4} \frac{1}{\sqrt{x}} dx = \frac{x^2}{2}\Big|_{1}^{4} + 2\int_{1}^{4} \frac{1}{2\sqrt{x}} dx = \frac{4^2}{2} - \frac{1^2}{2} + 2\int_{1}^{4} d\sqrt{x} = \frac{15}{2} + 2\sqrt{x}\Big|_{1}^{4} =$$
$$= \frac{15}{2} + 2\left(\sqrt{4} - \sqrt{1}\right) = \frac{15}{2} + 2(2-1) = \frac{15}{2} + 2 = \frac{19}{2} = 9.5.$$

Answer: 9.5.

Integrate with respect to x: $\int \sec x \cdot \tan x\ dx$.

Solution

Method 1 (substitution).

$$\int \sec x \cdot \tan x\ dx = \int \frac{\sin x}{\cos^2 x} dx = \left| u = \sec x, du = d\left(\frac{1}{\cos x}\right) = -\frac{-\sin x}{\cos^2 x} dx = \sec x \cdot \tan x\, dx \right| =$$

$= \int du = u + C = \sec x + C$, where C is an arbitrary real constant.

Method 2 (integration by parts).

$\int \sec(x) \cdot \tan(x)\ dx = \int \frac{\sin(x)}{\cos^2(x)} dx = \int \sin(x)\, d(\tan(x)) = |u = \sin(x), dv = d(\tan(x)), du =$
$\cos(x)\, dx, v = \tan(x)| = \sin(x)\tan(x) - \int \tan(x)\cos(x)\, dx = \frac{sin^2(x)}{\cos(x)} - \int \sin(x)\, dx =$

$$= \frac{sin^2(x)}{\cos(x)} + \cos(x) + C = \frac{sin^2(x) + cos^2(x)}{\cos(x)} + C = \frac{1}{\cos(x)} + C = \sec(x) + C,$$

where C is an arbitrary real constant.

Answer: $\sec x + C$.

Integrate with respect to x: $\int \csc 2x\, dx$.

Solution

First we use Trigonometric Identity, namely Reciprocal Identity: $\csc x = \frac{1}{\sin x}$. So, $\csc 2x = \frac{1}{\sin 2x}$.

$$\int \csc 2x\, dx = \int \frac{1}{\sin 2x} dx.$$

Method 1.

Let's multiply and divine the integrand, i.e. the function that is to be integrated, $\frac{1}{\sin 2x}$ by $\sin 2x$:

$$\int \frac{1}{\sin 2x} dx = \int \frac{\sin 2x}{\sin^2 2x} dx = \left| \begin{matrix} Pythadorean\ identity \\ \sin^2 2x = 1 - \cos^2 2x \end{matrix} \right| = \int \frac{\sin 2x}{1-\cos^2 2x} dx = -\frac{1}{2} \int \frac{d \cos 2x}{1-\cos^2 2x}$$
$$= |t = \cos 2x| = -\frac{1}{2}\int \frac{dt}{1-t^2} = \left| \frac{1}{1-t^2} = \frac{1}{(1-t)(1+t)} = \frac{1}{2}\left(\frac{1}{1-t} + \frac{1}{1+t}\right) \right| =$$

$$= -\frac{1}{4}\int \frac{dt}{1-t} - \frac{1}{4}\int \frac{dt}{1+t} = \frac{1}{4}\int \frac{dt}{t-1} - \frac{1}{4}\int \frac{dt}{t+1} = -\frac{1}{4} ln \left|\frac{t+1}{t-1}\right| + C = \frac{1}{4}\ln|1-t| - \frac{1}{4}\ln|1+t| + c$$
$$= \frac{1}{4}\ln\left|\frac{1-t}{1+t}\right| + c.$$

Thus,

$\int \csc 2x\, dx = \frac{1}{4}\ln\left|\frac{1-\cos 2x}{1+\cos 2x}\right| + c = \frac{1}{4}\ln\left|\frac{2sin^2(x)}{2cos^2(x)}\right| + c = \frac{1}{4}\ln|tan^2(x)| + c = \frac{1}{2}\ln|\tan(x)| + c$, where c is an arbitrary real constant.

Method 2.

$\int \frac{1}{\sin 2x} dx = \int \frac{dx}{2\sin(x)\cos(x)} = |divide\ the\ numerator\ and\ denominator\ by\ cos^2(x)| = \int \frac{\frac{dx}{cos^2(x)}}{\frac{2\sin(x)\cos(x)}{cos^2(x)}} =$
$\frac{1}{2}\int \frac{d(\tan(x))}{\tan(x)} = |y = \tan(x)| = \frac{1}{2}\int \frac{dy}{y} = \frac{1}{2}\ln|y| + c = \frac{1}{2}\ln|\tan(x)| + c$, where c is an arbitrary real constant.

Answer: $\boldsymbol{\frac{1}{2}\ln|\tan(x)| + c}$

Integrate with respect to x: $\int \sin x\ dx$.

Solution

$\int \sin x\ dx = -\cos x + C, C \in R.$

Answer : $\boldsymbol{-\cos x + C.}$

Integrate with respect to Q: $\int_0^{\frac{\pi}{3}} (2\sin Q - 5\cos Q) dQ$.

Solution

$$\int_0^{\frac{\pi}{3}} (2\sin Q - 5\cos Q) dQ = (-2\cos Q - 5\sin Q)\Big|_0^{\frac{\pi}{3}} = -2\cdot\frac{1}{2} - 5\cdot\frac{\sqrt{3}}{2} + 2 - 0 = 1 - \frac{5\sqrt{3}}{2} = \frac{2-5\sqrt{3}}{2}$$
$$\approx -3.33013.$$

Answer: $\boldsymbol{\frac{2-5\sqrt{3}}{2} \approx -3.33013.}$

Integrate with respect to v : $\int_{63}^{1} (4v - 2) dv$

Solution

Method 1

$$\int_{63}^{1}(4v-2)dv=\left(4\frac{v^2}{2}-2v\right)\Big|_{63}^{1}=\left(4\frac{1^2}{2}-2\cdot 1\right)-\left(4\frac{63^2}{2}-2\cdot 63\right)=-7812.$$

Method 2

$$\int_{63}^{1}(4v-2)dv=\frac{1}{4}\int_{63}^{1}(4v-2)d(4v-2)=|substitution\ t=4v-2, t(63)=250, t(1)=2|$$

$$=\frac{1}{4}\int_{250}^{2}tdt=\frac{1}{4}\frac{t^2}{2}\Big|_{250}^{2}=\frac{1}{8}(2^2-250^2)=-7812$$

Answer: -7812.

Integrate with respect to x: $\int_0^1(7-4x)^2dx$.

Solution

Method 1.

$$\int_{0}^{1}(7-4x)^2dx=\int_{0}^{1}(49-56x+16x^2)dx=49x-28x^2+\frac{16x^3}{3}|_0^1=\frac{79}{3}.$$

Method 2.

$$\int_{0}^{1}(7-4x)^2dx=-\frac{1}{4}\int_0^1(7-4x)^2d(7-4x)=|substitution\ t=7-4x, t(0)=7, t(1)=3|$$

$$=-\frac{1}{4}\int_7^3 t^2dt=-\frac{1}{4}\frac{t^3}{3}\Big|_7^3=-\frac{1}{12}(3^3-7^3)=\frac{316}{12}=\frac{79}{3}$$

Answer: $\frac{79}{3}$.