Answer on Question #52101 – Math – Integral Calculus

1 Find ∫ e dx

2 Find ∫ e2xdx

4 Find the ∫ tan3xsec3xdx

5 Find ∫ sec3xtanxdx

6 Find the integral with respect to x : ∫ cosxsinxdx

7 Find the integral with respect to x : ∫ (ex x)(ex 1)dx

8 Integrate with respect to x : ∫ dx

2

−1

x2

(x3 + 4)2

Solution

Let *C* be an arbitrary real constant.

1. $\int \boldsymbol{edx} = \boldsymbol{e} \int \boldsymbol{dx} = \boldsymbol{ex} + \boldsymbol{C};$

2. $\int \boldsymbol{e^{2x}dx} = \frac{1}{2} \int \boldsymbol{e^{2x}d(2x)} = |\boldsymbol{substitution\ t = 2x}| = \frac{1}{2} \int \boldsymbol{e^t dt} = \frac{1}{2}\boldsymbol{e^t} +$

$+\boldsymbol{C} = \frac{1}{2}\boldsymbol{e^{2x}} + \boldsymbol{C};$

4.

<u>Method 1.</u>

$$\int \boldsymbol{tan(3x)sec(3x)dx} = \int \frac{\boldsymbol{sin(3x)dx}}{\boldsymbol{cos^2(3x)}} = -\frac{1}{3} \int \frac{\boldsymbol{d(cos(3x))}}{\boldsymbol{cos^2(3x)}} = \left| \begin{matrix} \boldsymbol{substitution} \\ \boldsymbol{t = cos(3x)} \end{matrix} \right| =$$

$$= -\frac{1}{3} \int \boldsymbol{t^{-2}dt} = -\frac{1}{3} \cdot \left(\frac{\boldsymbol{t^{-1}}}{-1}\right) + \mathrm{C} = \frac{1}{3\boldsymbol{t}} + \boldsymbol{C} = \frac{1}{3\boldsymbol{cos(3x)}} + \boldsymbol{C} = \frac{1}{3}\boldsymbol{sec(3x)} + \boldsymbol{C};$$

Method 2.

$$\int tan(3x)sec(3x)dx = \int \frac{sin(3x)dx}{cos^2(3x)} = \frac{1}{3}\int sin(3x)\, d\big(tan(3x)\big)$$

$$= \left|integration\ by\ parts\ u = sin(3x), dv = d\big(tan(3x)\big)\right|$$

$$= \frac{1}{3}sin(3x)tan(3x) - \frac{1}{3}\int 3tan(3x)cos(3x)dx = \frac{1}{3}\cdot\frac{sin^2(3x)}{cos(3x)}$$

$$-\frac{1}{3}\int sin(3x)d(3x) = \frac{1}{3}\cdot\frac{sin^2(3x)}{cos(3x)} + \frac{1}{3}cos(3x) + C =$$

$$= \frac{1}{3cos(3x)}\Big(sin^2(3x) + cos^2(3x)\Big) + C =$$

$$= \left|sin^2(3x) + cos^2(3x) = 1\right| = \frac{1}{3cos(3x)} + C = \frac{1}{3}sec(3x) + C$$

5. $\int sec^3xtanxdx = \int \frac{sin(x)dx}{cos^4(x)} = -\int \frac{d(cos3x)}{cos^4(3x)} = \left|\begin{matrix}substitution\\ t = cos(3x)\end{matrix}\right| =$

$$= -\int \frac{dt}{t^4} = -\int t^{-4}dt = -\frac{t^{-3}}{-3} + C = \frac{1}{3t^3} + C = \frac{1}{3cos^3(3x)} +$$

$+C = \frac{1}{3}sec^3(3x) + C.$

6.

Method 1.

$\int cos(x)sin(x)dx = |sin(2x) = 2sin(x)cos(x)| = \frac{1}{2}\int sin(2x)dx = \qquad =$
$\frac{1}{4}\int sin(2x)d(2x) = |substitution\ t = 2x| = \frac{1}{4}\int sin(t)dt =$

$$= -\frac{1}{4}cos(t) + C_1 = -\frac{1}{4}cos(2x) + C_1.$$

Method 2.

$\int cos(x)sin(x)dx = \int sin(x)d\big(sin(x)\big) = |substitution\ t = sin(x)| =$

$$= \int tdt = \frac{t^2}{2} + C_2 = \frac{sin^2x}{2} + C_2.$$

Recall $cos(2x) = cos^2x - sin^2x = 1 - 2sin^2(x)$, therefore

$-\frac{1}{4}cos(2x) + C_1 = -\frac{1}{4}\left(1 - 2sin^2(x)\right) + C_1 = -\frac{1}{4} + \frac{sin^2x}{2} + C_1 = \frac{sin^2x}{2} + C_2,$

so we can assume $C_2 = C_1 - \frac{1}{4}$ and methods give the same answer.

7.

Method 1.

$\int(e^x - x)(e^x - 1)\ dx = \int(e^{2x} - xe^x - e^x + x)dx = \int e^{2x}dx - \int xe^x dx -$
$- \int e^x dx + \int x dx = \frac{1}{2}\int e^{2x}d(2x) - (x-1)e^x - e^x + \frac{1}{2}x^2 =$

$$= |substitution\ t = 2x| = \frac{1}{2}\int e^t dt - (x-1)e^x - e^x + \frac{1}{2}x^2 =$$

$$= \frac{1}{2}e^t - (x-1)e^x - e^x + \frac{1}{2}x^2 + C =$$

$= \frac{1}{2}e^{2x} - (x-1)e^x - e^x + \frac{1}{2}x^2 + C = \frac{(e^x - x)^2}{2} + C.$

We used the integrals $\int e^x dx = e^x + C$ and

$\int xe^x\,dx = \int x de^x = \left|\begin{matrix} integration\ by\ parts\ \ u = x, dv = de^x, \\ du = dx, v = e^x \end{matrix}\right| = xe^x -$
$- \int e^x dx = xe^x - e^x + C = (x-1)e^x + C.$

Method 2 .

$\int(e^x - x)(e^x - 1)\ dx = \int(e^x - x)(e^x - x)'\,dx = \int(e^x - x)d(e^x - x) =$
$|substitution\ t = (e^x - x),\ dt = (e^x - 1)dx| = \int t dt = \frac{t^2}{2} + C =$

$= \frac{(e^x - x)^2}{2} + C.$

8.

Method 1 (straightforward computation). Using the Newton-Leibniz axiom (or the second fundamental theorem of calculus) and table integral of powers, compute

$\int_{-1}^{2} x^2(x^3 + 4)^2 dx = \int_{-1}^{2}(x^8 + 8x^5 + 16x^2)dx =$

$$= \left(\frac{1}{9}x^9 + \frac{8}{6}x^6 + \frac{16}{3}x^3\right)\Big|_{x=-1}^{x=2} = \frac{512}{9} + \frac{256}{3} + \frac{128}{3} + \frac{1}{9} - \frac{4}{3} + \frac{16}{3} = 189.$$

<u>Method 2</u> (substitution). Using the Newton-Leibniz axiom (or the second fundamental theorem of calculus) and table integral of powers, compute

$$\int_{-1}^{2} x^2(x^3+4)^2 dx = \frac{1}{3}\int_{-1}^{2} (x^3+4)^2 d(x^3+4) =$$

$$= \left| \begin{matrix} substitution \\ t = x^3+4, t(-1) = 3, t(2) = 12 \end{matrix} \right| = \frac{1}{3}\int_3^{12} t^2 dt = \frac{1}{3}\cdot\frac{t^3}{3}\Big|_3^{12} = \frac{1}{9}(12^3 - 3^3) =$$

$$= 189.$$