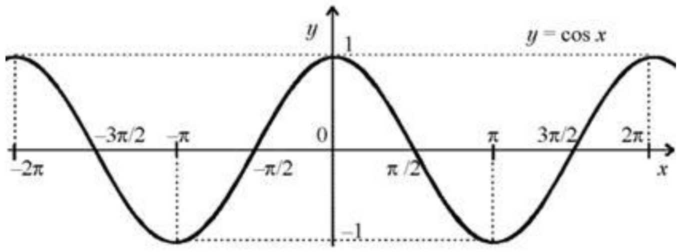
Answer on Question #51706 – Math – Integral Calculus

$\int_{-2\pi}^{\pi} |\cos x| dx = ?$

Solution

To find this definite integral, we must know subsets of $[-2\pi; \pi]$, where function $\cos x$ is negative and positive. We can see it on a picture below:

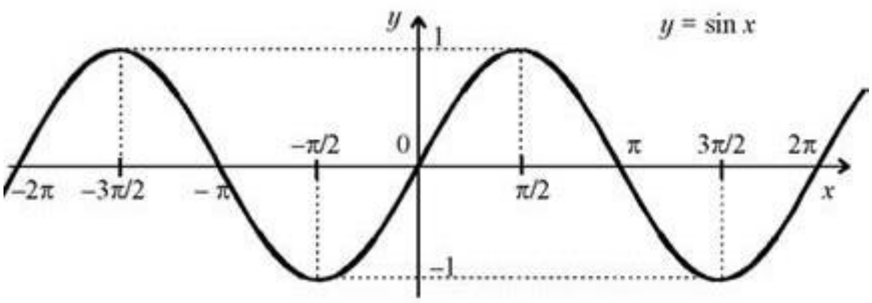


Now, using definition of the absolute value $|\cos x| = \begin{cases} \cos x, & \cos x \geq 0 \\ -\cos x, & \cos x < 0 \end{cases}$, we can rewrite our integral as follows:

$$\int_{-2\pi}^{\pi} |\cos x| dx = \int_{-2\pi}^{-3\pi/2} \cos x\, dx - \int_{-3\pi/2}^{-\pi/2} \cos x\, dx + \int_{-\pi/2}^{\pi/2} \cos x\, dx - \int_{\pi/2}^{\pi} \cos x\, dx =$$

$$= \sin x \Big|_{-2\pi}^{-\frac{3\pi}{2}} - \sin x \Big|_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} + \sin x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \sin x \Big|_{\frac{\pi}{2}}^{\pi} = \sin\left(-\frac{3\pi}{2}\right) - \sin(-2\pi) - \sin\left(-\frac{\pi}{2}\right) +$$
$$\sin\left(-\frac{3\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) - \sin(\pi) + \sin\left(\frac{\pi}{2}\right) = 1 - 0 - (-1) + 1 + 1 - (-1) - 0 +$$
$$+1 = 6.$$

The second fundamental theorem of calculus (or Newton-Leibniz axiom), the fact that the antiderivative of cosine is sine plus an integration constant, some values of sine function were applied there.



Answer:

$\int_{-2\pi}^{\pi} |\cos x| dx = 6.$