Answer on Question #51701 – Math – Integral Calculus

What is the definite integral of this $|x^2 - 3x - 10|$, limit is from -2 to 8?

Solution

We must calculate the following integral:

$\int_{-2}^{8} |x^2 - 3x - 10| dx.$

To do this, we need to know how function $x^2 - 3x - 10$ behave on segment $[-2; 8]$:

$x^2 - 3x - 10 = 0 \quad => \quad (x - 5)(x + 2) = 0.$

We see that this function takes negative values on $[-2; 5]$ and takes positive values on $[5; 8]$. Now, using definition of absolute value $|A| = \begin{cases} A, if\ A \geq 0 \\ -A, if\ A < 0 \end{cases}$, we can rewrite the integral:

$\int_{-2}^{8} |x^2 - 3x - 10| dx = \int_{-2}^{5} (-x^2 + 3x + 10) dx + \int_{5}^{8} (x^2 - 3x - 10) dx = \left(-\frac{x^3}{3} + \frac{3}{2}x^2 + 10x\right)\Big|_{-2}^{5} + \left(\frac{x^3}{3} - \frac{3}{2}x^2 - 10x\right)\Big|_{5}^{8} = \left(-\frac{5^3}{3} + \frac{3}{2}5^2 + 10 \cdot 5\right) - \left(-\frac{(-2)^3}{3} + \frac{3}{2}(-2)^2 + 10 \cdot (-2)\right) + \left(\frac{8^3}{3} - \frac{3}{2}8^2 - 10 \cdot 8\right) - \left(\frac{5^3}{3} - \frac{3}{2}5^2 - 10 \cdot 5\right) = \frac{293}{3}.$

Answer:

$\int_{-2}^{8} |x^2 - 3x - 10| dx = \frac{293}{3}$