Question #51654, Math, Matrix | Tensor Analysis | for completion

For the following matrix calculate the determinant:
{{5,3,7},{1,0,1},{9,6,4}}

Answer to Question#51654

Solution

The determinant is a scalar value that can be calculated from a square matrix.

$$A = \begin{pmatrix} 5 & 3 & 7 \\ 1 & 0 & 1 \\ 9 & 6 & 4 \end{pmatrix}$$

The standard way to compute the determinant of a square matrix is to apply the Laplace expansion along any one of its rows or columns. For instance, an expansion along the first row yields:

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot (-1)^{1+1} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12} \cdot (-1)^{1+2} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \cdot (-1)^{1+3} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} =$$

$$= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} =$$

$$= a_{11} \cdot (a_{22}a_{33} - a_{23}a_{32}) - a_{12} \cdot (a_{21}a_{33} - a_{23}a_{31}) + a_{13} \cdot (a_{21}a_{32} - a_{22}a_{31})$$

$$\det(A) = |A| = \begin{vmatrix} 5 & 3 & 7 \\ 1 & 0 & 1 \\ 9 & 6 & 4 \end{vmatrix} = 5 \cdot (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 6 & 4 \end{vmatrix} + 3 \cdot (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 9 & 4 \end{vmatrix} + 1 \cdot (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 9 & 6 \end{vmatrix} =$$

$$= 5 \cdot (0 \cdot 4 - 6 \cdot 1) - 3 \cdot (1 \cdot 4 - 1 \cdot 9) + 7 \cdot (1 \cdot 6 - 0 \cdot 9) = -30 + 15 + 42 = 27.$$

However the simplest way to compute the determinant of this matrix A is to use the Laplace expansion along the second row:

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{21} \cdot (-1)^{2+1} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{22} \cdot (-1)^{2+2} \cdot \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + a_{23} \cdot (-1)^{2+3} \cdot \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} =$$

$$= -a_{21} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{22} \cdot \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} - a_{23} \cdot \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} =$$

$$= -a_{21} \cdot (a_{12}a_{33} - a_{13}a_{32}) + a_{22} \cdot (a_{11}a_{33} - a_{13}a_{31}) - a_{23} \cdot (a_{11}a_{32} - a_{12}a_{31})$$

$$\det(A) = |A| = \begin{vmatrix} 5 & 3 & 7 \\ 1 & 0 & 1 \\ 9 & 6 & 4 \end{vmatrix} = 1 \cdot (-1)^{2+1} \begin{vmatrix} 3 & 7 \\ 6 & 4 \end{vmatrix} + \underbrace{0 \cdot (-1)^{2+2} \begin{vmatrix} 5 & 7 \\ 9 & 4 \end{vmatrix}}_{0} + 1 \cdot (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 9 & 6 \end{vmatrix} = -(3 \cdot 4 - 6 \cdot 7) - (5 \cdot 6 - 3 \cdot 9) =$$

$$= -(12 - 42) - (30 - 27) = 30 - 3 = 27$$

Answer: $\det(A) = 27$