Answer on Question #51643 – Math – Matrix | Tensor Analysis

Solve the system of simultaneous equations below using any of the matrix method

$$\begin{array}{ccccccc} 2x & + & 3y & - & z & = & 6 \\ 3x & + & y & + & 5z & = & 8 \\ x & + & 2y & + & z & = & 10 \end{array}$$

Solution

Cramer's rule will be applied to calculate x, y, z.

Coefficient matrix

$$\begin{pmatrix} 2 & 3 & -1 \\ 3 & 1 & 5 \\ 1 & 2 & 1 \end{pmatrix}$$

and answer column

$$\begin{pmatrix} 6 \\ 8 \\ 10 \end{pmatrix}$$

We have the left-hand side of the system with the variables (the "coefficient matrix") and the right-hand side with the answer values.

Let D be the determinant of the coefficient matrix of the above system, and let $D_x$be the determinant formed by replacing the x −column values with the answer-column values

$$D = \begin{vmatrix} 2 & 3 & -1 \\ 3 & 1 & 5 \\ 1 & 2 & 1 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 6 & 3 & -1 \\ 8 & 1 & 5 \\ 10 & 2 & 1 \end{vmatrix}$$

Similarly, D_y and D_z would then be the following:

$$D_y = \begin{vmatrix} 2 & 6 & -1 \\ 3 & 8 & 5 \\ 1 & 10 & 1 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 2 & 3 & 6 \\ 3 & 1 & 8 \\ 1 & 2 & 10 \end{vmatrix}$$

Evaluating each determinant, we get:

$$D = \begin{vmatrix} 2 & 3 & -1 \\ 3 & 1 & 5 \\ 1 & 2 & 1 \end{vmatrix} = |expand\ along\ the\ first\ column| = 2\begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ 1 & 5 \end{vmatrix}$$

$$= 2(1 \cdot 1 - 2 \cdot 5) - 3(3 \cdot 1 - 2 \cdot (-1)) + (3 \cdot 5 - 1 \cdot (-1))$$

$$= 2(1 - 10) - 3(3 + 2) + (15 + 1) = -18 - 15 + 16 = -17$$

$$D_x = \begin{vmatrix} 6 & 3 & -1 \\ 8 & 1 & 5 \\ 10 & 2 & 1 \end{vmatrix} = |expand\ along\ the\ first\ column|$$

$$= 6\begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} - 8\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} + 10\begin{vmatrix} 3 & -1 \\ 1 & 5 \end{vmatrix} = 6(1-10) - 8(3+2) + 10(15+1)$$

$$= -54 - 40 + 160 = 66$$

$$D_y = \begin{vmatrix} 2 & 6 & -1 \\ 3 & 8 & 5 \\ 1 & 10 & 1 \end{vmatrix} = |expand\ along\ the\ first\ column|$$

$$= 2\begin{vmatrix} 8 & 5 \\ 10 & 1 \end{vmatrix} - 3\begin{vmatrix} 6 & -1 \\ 10 & 1 \end{vmatrix} + \begin{vmatrix} 6 & -1 \\ 8 & 5 \end{vmatrix} = 2(8-50) - 3(6+10) + (30+8)$$

$$= -84 - 48 + 38 = -94$$

$$D_z = \begin{vmatrix} 2 & 3 & 6 \\ 3 & 1 & 8 \\ 1 & 2 & 10 \end{vmatrix} = |expand\ along\ the\ first\ column| = 2\begin{vmatrix} 1 & 8 \\ 2 & 10 \end{vmatrix} - 3\begin{vmatrix} 3 & 6 \\ 2 & 10 \end{vmatrix} + \begin{vmatrix} 3 & 6 \\ 1 & 8 \end{vmatrix}$$

$$= 2(10-16) - 3(30-12) + (24-6) = -12 - 54 + 18 = -48$$

Cramer's Rule says that $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, $z = \frac{D_z}{D}$,

that is,

$$x = \frac{66}{-17} = -3\frac{15}{17}$$

$$y = \frac{-94}{-17} = 5\frac{9}{17}$$

$$z = \frac{-48}{-17} = 2\frac{14}{17}$$

Answer: $\{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\} = \left\{-\mathbf{3}\frac{\mathbf{15}}{\mathbf{17}}, \mathbf{5}\frac{\mathbf{9}}{\mathbf{17}}, \mathbf{2}\frac{\mathbf{14}}{\mathbf{17}}\right\}$