Answer on Question #51629 – Math – Integral Calculus

What is the integration of $\frac{1}{\sqrt{(x-1)(x-2)(x-3)}}$

Solution

Let

$$I(x) = \int \frac{1}{\sqrt{(x-1)(x-2)(x-3)}} dx.$$

This integral cannot be expressed in elementary functions.

We use *Wolfram Mathematica online integrator* and find

$$I(x) = \frac{\left(2i\sqrt{\left(\frac{1}{x-3}+1\right)}\sqrt{\left(\frac{2}{x-3}+1\right)}(x-3)^{\frac{3}{2}} \cdot F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-3}}\right)\middle|2\right)\right)}{\sqrt{(x-1)(x-2)(x-3)}},$$

where i is imaginary unit, $\sinh^{-1}(x)$ is inverse hyperbolic sine, $F(x|m)$ is elliptic integral of the first kind.

We can simlify it to

$$I(x) = \frac{\left(2i\sqrt{\left(\frac{x-2}{x-3}\right)}\sqrt{\left(\frac{x-1}{x-3}\right)}(x-3)^{\frac{3}{2}} \cdot F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-3}}\right)\middle|2\right)\right)}{\sqrt{(x-1)(x-2)(x-3)}} = 2i \cdot F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-3}}\right)\middle|2\right).$$

Answer: $\mathbf{2i \cdot F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-3}}\right)\middle|2\right)}$.