Answer on Question #51628 – Math – Functional Analysis

Question. If $f : X \to Y$ and $g : Y \to Z$ then the domain of $g \circ f$ is X and co-domain is Z. What is the domain and co-domain of $f \circ g$, $f \circ f$, $g \circ g$, here in this case?

Solution. Notice that the composition of map $g \circ f$ is possible only when the codomain (the image) of f coincides with the domain of g. In other words, we could write the composition of arrows:

$$g \circ f : X \xrightarrow{f} Y \xrightarrow{g} Z.$$

However, in general, neither of the compositions $f \circ g$, $f \circ f$, $g \circ g$ are possible.

Nevertheless, if $f \circ g$ is defined, then we should have that $Z = X$:

$$f \circ g : Y \xrightarrow{g} Z \ = \ X \xrightarrow{f} Y.$$

In this case Y is the domain and co-domain of $f \circ g$.

Similarly, if $f \circ f$ is defined, then we should have that $X = Y$:

$$f \circ f : X \xrightarrow{f} Y \ = \ X \xrightarrow{f} Y.$$

In this case $X = Y$ is the domain and co-domain of $f \circ f$.

By the same reason, if $g \circ g$ is defined, then we should have that $Y = Z$:

$$g \circ g : Y \xrightarrow{g} Z \ = \ Y \xrightarrow{g} Z.$$

In this case $Y = Z$ is the domain and co-domain of $f \circ f$.

Answer.

1) The composition $f \circ g$ is defined only for $Z = X$, and in this case Y is the domain and co-domain of $f \circ g$.

2) The composition $f \circ f$ is defined only for $X = Y$, and in this case $X = Y$ is the domain and co-domain of $f \circ f$.

3) The composition $g \circ g$ is defined only for $Y = Z$, and in this case $Y = Z$ is the domain and co-domain of $g \circ g$.