

Answer on Question #51588 – Math – Calculus

Find the geometric series whose sum is 1 and first term is 0.4

Given

$a_1=0.4$ - the first term of geometric series

$S=1$ - the sum of geometric series

Find common ratio r and geometric series $a_1, a_2, a_3, a_4, \dots$

Solution

The **sum of a certain number of terms** of a **geometric sequence**:

$$S_n = \sum_{i=1}^n a_i = \frac{a_1(1 - r^n)}{1 - r}$$

where S_n is the sum of n terms (n^{th} **partial sum**), a_1 is the first term, r is the common ratio.

The n^{th} **term** of a **geometric sequence**:

$$a_n = a_1 r^{n-1}$$

where a_1 is the first term of the sequence, r is the common ratio, n is the number of the term.

If $-1 < r < 1$, the sum S_n converges to the geometric series, where the sum is given by

$$S = \frac{a_1}{1-r} \tag{1}$$

Solve equation (1) for r :

$$1 = \frac{0.4}{1-r},$$

$$1-r = 0.4,$$

$$r = 1-0.4,$$

$$r = 0.6$$

Then compute

$$a_2 = a_1 r^1 = 0.4 \cdot 0.6 = 0.24, a_3 = a_1 r^2 = 0.4 \cdot 0.6^2, a_4 = a_1 r^3 = 0.4 \cdot 0.6^3, \dots$$

$$a_n = a_1 r^{n-1} = 0.4 \cdot 0.6^{n-1}$$

In this case the geometric series is **0.4+0.24+0.144+0.0864+.....**

If $r > 1$ or $r < -1$ the sum S_n diverges, solution is valid only when a partial sum and value of n are known.