

Answer on Question #51526 – Math – Statistics and Probability

Question

The average monthly electric bill of a random sample of 100 residents of a city is \$90 with a standard deviation of \$24. Construct a 95% confidence interval for the mean monthly electric bills of all residents.

Solution

$$n = 100$$

$$\bar{X} = 90$$

$$\sigma = 24$$

$$\gamma = 95\% \Rightarrow \alpha = 5\% = 0.05$$

$$S^2 = \frac{n}{n-1} \sigma^2$$

$$S = \sqrt{\frac{n}{n-1}} \sigma = \sqrt{\frac{100}{99}} 24 = 24.12$$

$$P\left(\bar{X} - t_{1-\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{1-\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}\right) = 1 - \alpha$$

where $t_{1-\frac{\alpha}{2}, n-1} = 1 - \frac{\alpha}{2}$ -level quantile of Student's t -distribution with $n-1$ degrees of freedom. $t_{1-\frac{0.05}{2}, 100-1} = 1.984$. Then

$$t_{1-\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}} = 1.984 \frac{24.12}{\sqrt{100}} = 4.785408, \text{ so}$$

$$90 - 4.785408 \leq \mu \leq 90 + 4.785408$$

$$85.215 \leq \mu \leq 94.785$$

is 95% confidence interval for the mean monthly electric bills of all residents.