Answer on Question #51525 – Math – Statistics and Probability

Question

The average monthly electric bill of a random sample of 100 residents of a city is $90 with a standard deviation $24. Construct a 90% confidence interval for the mean monthly electric bill of all residents.

Solution

We have $n = 100;\ \bar{x} = 90;\ \sigma = 24; 1 - \alpha = 0.9 \Rightarrow \alpha = 0.1$.

Using the Student's table we find $t(\alpha, n - 1) = t(0.1, 99) = 1.6604$.

Since $\sigma = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(x_j - \bar{x})^2} = 24$ then

$\sum_{j=1}^{n}(x_j - \bar{x})^2 = 576 \cdot 100 = 57600$ and "corrected" standard deviation is equal to

$s = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_j - \bar{x})^2} = \sqrt{\frac{57600}{99}} \approx 24.12$.

The required confidence interval has the following form:

$$\bar{x} - t(\alpha, n-1)\frac{s}{\sqrt{n}} < a < \bar{x} + t(\alpha, n-1)\frac{s}{\sqrt{n}} \Leftrightarrow a \in \left(90 - 1.6604 \cdot \frac{24.12}{10};\ 90 + 1.6604 \cdot \frac{24.12}{10}\right)$$

$\Leftrightarrow a \in (85.995; 94.005)$.

Answer: $(85.995; 94.005)$.