Answer on Question #51514 – Math – Differential Geometry

find a vector parametrization v(t),t (0,3) of the path(or loop) ABCA where A(1,0,1),B(1,1,0),C(0,1,1)

Solution

Path *ABCA* consists of lines *AB, BC, CA*.

Apply the parametric form of the equation of a line $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$, where (x, y, z) is any point on the line, (x_0, y_0, z_0) is a fixed point on the line and $\vec{l} = \langle a, b, c \rangle$ is some vector that is parallel to the line.

Line *AB* has $(x_0, y_0, z_0) = (x_A, y_A, z_A) = (1,0,1)$,

$\vec{l} = \overrightarrow{AB} = \langle x_B - x_A, y_B - y_A, z_B - z_A \rangle = \langle 1 - 1,1 - 0,0 - 1 \rangle = \langle 0; 1; -1 \rangle$.

If $t = 0$ then $(x, y, z) = (x_A, y_A, z_A) = (1,0,1)$; if $t = 1$ then $(x, y, z) = (x_B, y_B, z_B) = (1,1,0)$.

Coordinate $x = 1$ is constant on the line *AB*.

Thus, a vector parametrization of *AB* is $(x, y, z) = (1, t, 1 - t)$, $0 \le t \le 1$.

Line *BC* has $(x_0, y_0, z_0) = (x_B, y_B, z_B) = (1,1,0)$,

$\vec{l} = \overrightarrow{BC} = \langle x_C - x_B, y_C - y_B, z_C - z_B \rangle = \langle 0 - 1,1 - 1,1 - 0 \rangle = \langle -1; 0; 1 \rangle$.

If $t = 0$ then $(x, y, z) = (x_B, y_B, z_B) = (1,1,0)$; if $t = 1$ then $(x, y, z) = (x_C, y_C, z_C) = (0,1,1)$.

Coordinate $y = 1$ is constant on the line *BC.*

Thus, a vector parametrization of *BC* is $(x, y, z) = (1 - t, 1, t)$ $0 \le t \le 1$.

Line *CA* has $(x_0, y_0, z_0) = (x_C, y_C, z_C) = (0,1,1)$,

$\vec{l} = \overrightarrow{CA} = \langle x_A - x_C, y_A - y_C, z_A - z_C \rangle = \langle 1 - 0,0 - 1,1 - 1 \rangle = \langle 1; -1; 0 \rangle$.

If $t = 0$ then $(x, y, z) = (x_C, y_C, z_C) = (0,1,1)$; if $t = 1$ then $(x, y, z) = (x_A, y_A, z_A) = (1,0,1)$.

Coordinate $z = 1$ is constant on the line *CA*.

Thus, a vector parametrization of *CA* is $(x, y, z) = (t, 1 - t, 1)$, $0 \le t \le 1$. In figure colour of *AB* is blue, colour of *BC* is yellow, colour of *CA* is green.