

Answer on Question #51512 - Math - Differential Geometry

Find the Bezier curve where the control points are $P(2,3), P_1(2,0), P_2(3,1), P_3(4,4)$

Solution

Since there are four control points, then we have cubic Bezier curve. Thus the explicit form of the Bezier curve is given by

$$B(t) = (1-t)^3 P + 3(1-t)^2 t P_1 + 3(1-t)t^2 P_2 + t^3 P_3, \quad t \in [0,1].$$

Hence

$$\begin{aligned} B(t) &= (1-t)^3 \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 3(1-t)^2 t \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 3(1-t)t^2 \begin{pmatrix} 3 \\ 1 \end{pmatrix} + t^3 \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 2(1-t)^3 + 6(1-t)^2 t + 9(1-t)t^2 + 4t^3 \\ 3(1-t)^3 + 3(1-t)t^2 + 4t^3 \end{pmatrix} = \\ &= \begin{pmatrix} 2 - 6t + 6t^2 - 2t^3 + 6t - 12t^2 + 6t^3 + 9t^2 - 9t^3 + 4t^3 \\ 3 - 9t + 9t^2 - 3t^3 + 3t^2 - 3t^3 + 4t^3 \end{pmatrix} = \begin{pmatrix} 2 + 3t^2 - t^3 \\ 3 - 9t + 12t^2 - 2t^3 \end{pmatrix}, \quad t \in [0,1]. \end{aligned}$$

Answer: $B(t) = (2 + 3t^2 - t^3)\bar{i} + (3 - 9t + 12t^2 - 2t^3)\bar{j}, \quad t \in [0,1].$