Answer on Question #51466 – Math – Algorithms | Quantitative Methods

The bacteria concentration in a reservoir varies as $e^t - \frac{t^3}{6} - e^{0.3t} - \frac{t^2}{2} - t$ where t is the time in seconds. Use the Newton-Raphson method to estimate the time required for the bacteria concentration to reach 1 (correct up to 2 decimal places)

Solution.

We need to find the first positive root of the following equation:

$$f(t) = e^t - e^{0.3t} - \frac{t^3}{6} - \frac{t^2}{2} - t - 1 = 0;$$

Use the Newton-Raphson method. Firstly compute $f'(t)$:

$$f'(t) = e^t - 0.3e^{0.3t} - \frac{t^2}{2} - t - 1 = f(t) + 0.7e^{0.3t} + \frac{t^3}{6};$$

So:

$$t_0 = 2;$$

$$t_1 = t_0 - \frac{f(t_0)}{f'(t_0)};$$

$$f(t_0) = e^2 - e^{0.6} - \frac{19}{3} \approx -0.766;$$

$$f'(t_0) = f(t_0) + 0.7e^{0.6} + \frac{4}{3} \approx 1.843;$$

So:

$$t_1 = 2 + \frac{0.766}{1.843} \approx 2.416;$$

$$f(t_1) \approx 0.452;$$

$$f'(t_1) \approx 4.247;$$

So:

$$t_2 = 2.416 - \frac{0.452}{4.247} \approx 2.309;$$

$$f(t_2) \approx 0.039;$$

$$f'(t_2) \approx 3.49;$$

So:

$$t_3 = 2.309 - \frac{0.039}{3.49} \approx 2.298;$$

$$f(t_3) \approx 0.001;$$

$$f'(t_3) \approx 3.418;$$

So:

$$t_4 = 2.298 - \frac{0.001}{3.418} \approx t_3.$$

Answer: 2.298