Answer on Question #51465 – Math – Algorithms | Quantitative methods

The time versus velocity data of a particle is given in the table below. Use Lagrange's interpolation formula to find the distance moved by a particle and its acceleration at the end of 3 seconds.

t: 0, 1, 2, 5

v: 2, 3, 12, 147

Solution

Lagrange's interpolation formula:

$$\boldsymbol{L(x) = \sum_{i=0}^{n} f_i \prod_{\substack{0\le k\le n \\ k\ne i}}^{n} \frac{x - x_k}{x_i - x_k}}$$

In this problem:

Velocity

$$\boldsymbol{L(t) = \sum_{i=0}^{3} v_i \prod_{\substack{0\le k\le n \\ k\ne i}}^{3} \frac{t-t_k}{t_i-t_k} = 2\frac{(t-1)(t-2)(t-5)}{(-1)*(-2)*(-5)} + 3\frac{t(t-2)(t-5)}{1*(-1)*(-4)} + 12\frac{t(t-1)(t-5)}{2*1*(-3)} +}$$

$$\boldsymbol{+147\frac{t(t-1)(t-2)}{5*4*3} = t^3 + t^2 - t + 2.}$$

The distance moved by a particle at the end of 3 seconds

$$\textbf{So, } \boldsymbol{S(3) = \int_0^3 v(t)dt = \left(\frac{1}{4}t^4 + \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t\right)\Big|_{t=0}^{t=3} = \left(\frac{1}{4}3^4 + \frac{1}{3}3^3 - \frac{1}{2}3^2 + 2\cdot 3\right) -}$$

$$\boldsymbol{-\left(\frac{1}{4}0^4 + \frac{1}{3}0^3 - \frac{1}{2}0^2 + 2\cdot 0\right) = \frac{123}{4} = 30.75.}$$

Acceleration

$$\boldsymbol{a = \frac{dv}{dt} = 3t^2 + 2t - 1;}$$

Acceleration at the end of 3 seconds

$$\boldsymbol{a(3) = 32.}$$